DEVELOPMENT OF TABLES FOR USE IN CELESTIAL NAVIGATION ON THE LUNAR SURFACE

20274

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Sec. 1 INTRODUCTION AND PRELIMINARY EVALUATION

The accuracy to which the positions can be determined using techniques of celestial navigation depends to a great extent on the accuracy of the navigation tables which in turn depend on astronomical constants and pameters and the basic ephemeris data. The specific task of this contract has been to generate a computer program in the ALGOL programming language which produces ephemeris and almanac data for use on the lunar surface. However, because the quantities are going to be computed on an electronic computer, it has been possible to treat the problem in complete generality. Thus, with only relatively minor modifications, the Lunar Astronomical Navigation System (LANS) can be applied to any navigation situation, including the earth. If these computations were to be derived strictly for the moon, several simplifying assumptions could be made. However, such a program would have to be rewritten, for example, if an ephemeris for Mars or for Mariner IV were desired. The lunar navigation system (LANS) requires only that the appropriate data be read in and some of the internal constants adjusted. Otherwise no major revisions are necessary. Neither is accuracy sacrificed because the methods used are the same, in most instances, as those used by observatories in the generation of terrestrial tables and almanacs.

It must be said that the methods used in LANS are completely conventional. We have adopted these methods to our own particular needs and brought them together into a consistent system. Thus, LANS represents a logical extention of prodigious work which has gone into the production of the terrestrial tables

The Astronomical Ephemeris (A.E.) and The American Ephemeris and Nautical Almanac (A.E.N.A.) And certainly the task of devising LANS would have been more difficult if the excellent reference volume Explanatory Supplement to the A.E. and A.E.N.A. had not been available. This text is invaluable to those intimately involved in astrometry. Reference to this will be made frequently throughout this report.

During the course of our investigation it came to our attention that we were not the only ones interested in navigational ephemerides for use on the moon. Gelins (1965) in a review appearing in the Foreign Science Bulletin describes in some detail an ephemeris for use on the lunar surface published in the U.S.S.R. The methods used differ in several respects from our own. Some of the differences are basic so the more important are listed below. The Russian tables were developed by Yakovkin, Dmenko and Miz:

1) The Russian table uses libration constants computed on a flattening value f = 0.82, we can input constants interpolated over a range of f values. For sample calculations we have chosen f = 0.63 as indicated by an extensive review by Goudas (1965).

2) The Russian method uses approximate corrections to terrestrial ephemerides and hence are applicable only on the moon. They may not give high accuracy in some cases

3) The Russian tables are computed assuming a uniform mean lunar sidereal time defined as a specific number of terrestrial sidereal days. Analogous lunar "hours, minutes, and seconds" were also defined. Since lunar travelers will almost certainly have a knowledge of terrestrial time and use a clock running on terrestrial mean solar time, Ephemeris, Universal

or other mean solar time is the most convenient for navigation tables. The most accurately determined uniform time is ephemeris time (E.T.) and our table generation program uses this as the reference scale. If the output is desired in mean terrestrial solar time (which is non-uniform) an option is available which will enable interpolation to universal time (U.T.), if the difference between E.T. and U.T. is known or can be predicted to a fair degree of accuracy.

4) The Russian tables are apparently computed using Brown's theory of lunar motion. The LANS system uses the lunar positions as provided by the U. S. Naval Observatory. These are based on a recently improved lunar theory and are considered accurate enough for timekeeping purposes.

Although we have considerable confidence in our programs, there are certain areas in which our present knowledge is limited. Also, it is desireable to actually test the lunar libration constants by direct observation. For this purpose we have proposed a contract extension dealing specifically with these problems. Some of the problems encountered when the programs are to be used for generating data of geodetic precision are:

1) size of free-libration terms
2) long-term periodic libration terms
3) incomplete knowledge of local gravity anomalies after "geoid" corrections have been applied
4) incomplete knowledge of center-of-mass coordinates of "geoid" center.

Computer softwave developed or modified under this contract are:

1) LANS BASIC TABLE GENERATOR
2) LANS ORBIT COMPUTATION ROUTINE
3) LANS EMPIRICAL DATA PROCESSOR
4) ORTHOGONAL FUNCTION FITTING ROUTINE
5) POWER SERIES LEAST-SQUARES FITTING ROUTINE
6) JULIAN DATE - CALENDAR DATE CONVERTER
7) CENTER-OF-MASS GENERATOR.

Sec. 2 SPHERICAL ASTRONOMY AND NAVIGATION

In the early days of seafaring, the science of astronomy and the art of navigation developed together. However, as the need for higher accuracy in timekeeping and positions became more widespread, astronomy and navigation parted company. National observatories and international commitees were set up to generate fundamental astronomical data. These could be used by astronomers directly, but were not suitable for use in navigation. Therefore, navigation tables had to be generated separately and the task was usually given to an agency other than a national observatory or almanac office.

Celestial navigation on the surface of the earth does not require very high precision. (The demands of positional astronomy require an accuracy of an order of magnitude higher than navigation). Traverses from one area to another is all that is necessary. Within these areas navigation may be done by dead-reckoning (using landmarks and high accuracy maps), triangulation, LORAN (or other passive electronic system) or radar. But these more modern methods are successful because of the extensive astronomical, geophysical, and geodetic investigations which have been carried out over large areas of the earth and over long periods of time.

But a detailed knowledge of astronomy or geophysics is not necessary to do celestial navigation provided the appropriate tabular data is available. In fact, one can (and does) teach celestial navigation to people with only a high school mathematics background. The same can be said for the elementary parts of spherical astronomy. Now when celestial navigation from bodies other than the earth is considered, one should try to define reference and coordinate systems completely analogous to the terrestrial case. Thus, standard methods tested on the earth can be applied successfully elsewhere with a minimum of modification.

Although a user of an "astronautical almanac" might not be interested in the complex calculations of the ephemeris data, others, particularly those who wish to adapt our program to their own needs, should be familiar with the basic astronomical concepts presented here. For a detailed account reference should be made to the appropriate chapter or section of the Explanatory Supplement to the Astronomical Ephemeris and the American Ephemeris and Nautical Almanac. Except where noted definitions conform with those in the Supplement and the recommendations of the Commission 4 of the International Astronomical Union, Sec. 2.1 Coordinate Systems (Chapters 1 and 2 in the Explanatory Supplement). When a solid body rotates in the absence of external forces, the initial conditions determine the direction of the rotation axis. A plane perpendicular to this axis is called the equator. The projection of the rotation axis on the celestial sphere is called a celestial pole. If the body is gravitationally connected with only one other body, the conservation of angular momentum (assuming only central force) requires that the orbits of the two be contained in the same plane. This orbital plane intersects the equatorial plane in a line called the line of nodes. The great circle representing the projection of the equatorial plane on the celestial sphere is called the celestial equator. The great circle representing the projection of the orbital plane of the earth is called the ecliptic. The great circle representing the projection of the orbital plane of any other object is usually not given a special name. Likewise, the term "equatorial coordinates" usually is taken to refer to coordinates relative to the earth's equator. The ascending node of the ecliptic relative to an object equator is the descending node of the equator relative to the ecliptic and is used as a prime coordiante direction. The north pole of the ecliptic is the projection on the celestial sphere of the perpendicular to the earth's orbit. This is given by a right-hand rule with the thumb in the direction of the pole, the forefinger in the direction of the velocity, and the middle finger in the direction of the center of mass. The north celestial pole is defined by a similar right-hand rule with the forefinger in direction of rotation of the body on its axis. The hemisphere (on the celestial sphere) between the north pole and the equator is called, obviously, the northern celestial hemisphere. That between the ecliptic and the northern ecliptic pole can be called the northern ecliptic hemisphere. Now the ascending node is the node of one plane ("A") relative to another ("B") where a particle, confined to plane "A" moves according to a right-hand rule, will travel from the southern hemisphere of plane "B" to the northern hemisphere of plane "B". The descending node requires motion from "northern" to "southern" motion.

The ascending node of the ecliptic on the earth's equator is given several special names: the first point of Aries, the vernal equinox, the point of Aries, or simply the equinox. If a set of rectangular axes are set up relative to the earth's equator, they are called Rectangular Equatorial Coordinates. The Z-axis points northward, the X-axis points to the point of Aries, and the Y-axis points such that $\overline{X} \times \overline{Y} = \overline{Z}$, i.e. right-hand system.

If the sun and the earth were only bodies in the solar system, the ecliptic and equatorial systems would be equally stable reference systems. (For observations from a planetary surface, an equatorial reference is more convenient than an ecliptic one, since in a spherical-equatorial system, angular quantities directly analogous to terrestrial longitude and latitude may be used to describe positions on the celestial sphere.) However, this is not the case. The moon and planets exert torques on rotational axis of the earth, as well as its orbital plane. Thus, neither the ecliptic nor the equator represent invarient plane. There is, however, an appropriate reference plane called, appropriately, the <u>Invarient Plane</u>. At any specific instant of time, the ecliptic and equatorial planes can be defined relative to the invarient plane. Likewise, the ascending node of the ecliptic, the equinox, is specified uniquely at each time. Frequently one simply speaks of "the equinox date" when actually the equinox, ecliptic, and equator are meant to be defined. The equinox and one plane (either ecliptic or equator) uniquely specify a fixed (prime, fundamental, invarient) reference system, if an epoch is defined. Thus, it is more convenient to chose an equinox near the dates concerned rather than be bothered with the transformations to, and from, the invarient plane.

The differences between the ecliptic and the invarient plane are always small ($\sim 0\overset{''}{.}5$ of arc) with periods of oscillation on the order of millions of years. Hence the term "ecliptic" is often used to mean the invarient plane. Because of the large perturbing action of the moon, the earth's equatorial plane undergoes much larger changes with somewhat shorter periods of oscillation. Thus, in order to obtain the coordinates of an object relative to a <u>moving</u> reference axis (of the earth's daily rotation, for instance) two correction terms are needed.

<u>Precession</u> is the component of axial motion which has a constant rate of change when projected on the celestial sphere. <u>Nutation</u> includes all

the remaining terms (both long and short period) necessary to describe the residuals of axial motion once precession effects have been removed. If the object is reasonably rapidly rotating, precession and nutation can be easily separated.

However, for slowly rotating objects, such as the moon, it is much easier to treat precession and nutation as a single perturbation of the axes called <u>physical libration</u>.

The concepts discussed so far have assumed that there is an infinite signal velocity, but this is not so. Coordinate definitions must take these into account. Because the velocity of light is finite, the direction from which a light ray is received by an object moving relative to an inertial frame is different from that "seen" by an object at rest in the same frame and at the same position. This effect is called the <u>aberration of light</u>. It results because, a coordinate system at the center of the earth, even if not rotating, is not inertial and a correction for abberation, in principle, reduces the system to an equivalent inertial frame. (It should be noted that correction for aberration only makes the system inertial in a <u>relative</u> sense. It is not possible to find an absolute inertial system. Thus, in the corrections that follow, one obtains positions as seen for a coordinate system at the center-of-mass of the solar system. This is called the <u>local standard of rest</u>. For interstellar navigation, a system which takes into account the velocity of the <u>stars relative</u> to the local standard of rest must be used or <u>incorrect</u> rectangular coordinate positions will be obtained.)

It is convenient to consider aberration of light from two standpoints. If the stars are considered at rest relative to the center-of-mass of the solar system (see parenthetical statement above), all positions can be reduced using a correction term which involves only the velocity of the observer about the center-of mass. This correction is called the stellar aberration term. This correction is applicable to situations where only a comparison with the star background is needed. If a reduction to actual coordinates is necessary, the total aberration, (frequently called the planetary aberration) must be taken into account. It consists of two contributions. First if the observer is moving relative to an object at rest in the inertial system, a contribution of stellar aberration exists for the observed position. Likewise, if the object were moving and the observer is stationary (relative to the center-of-mass), the object would "see" a stellar aberration of the conventional type. If both the observer and the object have motions relative to the center-of-mass, the two stellar aberrations combine and give rise to the planetary aberration.

There is another way of looking at planetary aberration. If the earth is regarded as fixed and a planet moves relative to it, the planet will have moved during the time it takes the light to reach the earth. Thus, the apparent position of the planet corresponds to where the planet was at $t_o - \tau$ (where τ is the light-time), not where it is at t_o. The difference in angular position between t_o and $(t_o - \tau)$ is the planetary aberration. Now that axial perturbations and aberrations have been briefly discussed, we return to the discussion of coordinate systems.

A number of special coordinate origins exist.

a) Topocentric - coordinates centered at an observer. In LANS, topocentric is reserved for coordinates centered on an observer who is located on the surface of a body with an independently defined gravitational field.

b) Stationocentric - coordinates centered at an observer's position, particularly an observer who is not located on the surface of a body with an independently defined gravitational field.

c) Selenocentric - center of moon.

d) Geocentric - center of the earth.

e) Barycentric - center of mass coordinates (body reference not specified.

f) Heliocentric - center of the sun.

Two types of coordinates are used predominately in astronomy - spherical or rectangular.

There are three main kinds of positions.

a) Geometric - the actual, physical positions of the objects (as required by the theory of gravitation in the case of planets).

b) Apparent - positions at which objects appear to be as a result of aberration of light.

c) Astrometric - apparent positions which have been corrected so that the object will be referenced to the same coordinate system as used in star catalogues. The correction term is the stellar aberration minus a small, nearly constant term.

$$M_o = A - (R - E)$$

where A is the apparent place, M_o is the mean catalogue place, R is the complete stellar aberration term, E is the small correction term. If λ is the ecliptic longitude and β the ecliptic latitude, then the correction E terms are

$$\Delta\lambda = Ke \sec \beta \cos (\omega - \lambda)$$
$$\Delta\beta = Ke \sin \beta \sin (\omega - \lambda)$$

where

$$K = 20''.47$$
$$e = 0.01675 - 0.00004T$$
$$\omega = 101^\circ.22 + 1^\circ.72T$$

where T is the number of Julian centuries (36500 days = one Julian century) since 1900.0. Stellar aberration is contributed from three sources:

1) Annual aberration - aberration due to the motion of the earth about the sun. Before 1960 corrections for annual aberration were made assuming a circular orbit for the earth. After 1960, corrections took into account the eccentricity of the earth's orbit.

2) Diurnal aberration - aberration due to the motion of the earth on its axis. This is a topocentric correction.

3) Secular aberration - aberration due to uniform motion of the star relative to the center-of-mass. This aberration is equal to the light-time of the star times its apparent proper motion. It is customary, as noted earlier, to omit the secular term in normal astronomical positions since the required information for evaluation of the term is available only for a small number of stars. If interstellar navigation is contemplated this term must be evaluated before transforming to geometric coordinates.

Refraction is a topocentric correction which must be applied if a reasonably dense atmosphere is present. Since the moon possesses no detectable "permanent" atmosphere, LANS does not include corrections for refraction. These corrections when needed are most conveniently applied to the observations not the ephemeris. In the computation of rise and set times some modification for refraction will be required.

Sec. 2.2 Time and Timekeeping (Chapter 3 of the Explanatory Supplement)

In this section, a brief summary of the types of times and their importance to the ephemeris and navigation problem. One may get very involved with the details of time determination, but this is not the purpose of this section; only fundamentals will be presented.

In the earlier days of astronomy it was thought that the rotation rate of the earth was constant. Over the years, astronomers have come to realize that the rotation of the earth is much less constant than the Keplerian motion of the earth and planets around the sun. The dynamics of the solar system thus provide a set of periodicities which are much more accurate for timekeeping than the earth's rotation. Ephemeris time is a time-scale which is the independent argument of gravitational motion. It is considered a uniform time and so is naturally used as the argument for ephemerides. It is independent of a planet's rotation and hence cannot be used, if the highest accuracy is desired, to predict topocentric phenomena such as transits, rising and setting times unless the rotation rate is constant.

The hour angle (whether local or zero longitude) of the vernal equinox is known as sidereal time. When ephemeris time is used to generate any topocentric tables, the sidereal time obtained is Ephemeris Sidereal Time; transits occur at the Ephemeris Meridian; and the longitude of the observer is noted as Ephemeris Longitude. Also note that,

LOCAL SIDEREAL TIME = GREENWICH (ZERO LONGITUDE) SIDEREAL TIME - LONGITUDE.

Apparent sidereal time is the hour angle of the true equinox, while mean sidereal time is the hour angle of the mean equinox of date.

Time which depends on the rotation of the earth is called Universal Time. Unlike Ephemeris Time it is subject to variation. Universal time is a type of Mean Solar Time. If U.T. is used as the independent variable, the sidereal time is called Universal Sidereal Time; the time reference point is the Universal Mean Sun; transits occur at the Universal Meridian; and the longitude of the observer is called Universal Longitude. For purposes of navigation either the "ephemeris" or "universal" times may be used.

For purposes of geodesy, the distinction must be made. In the LANS program, the basic argument is ephemeris time. If the difference between Ephemeris Time and Universal Time (ΔT) is known, then subtabulation routines are available. However, Universal Time is only a convenience for terrestrial observers, since lunar sidereal time, as defined later, uses ephemeris time as the independent variable as it logically should - lunar rotation, after all, is reasonably independent of the earth rate. The coordinates produced using E. T. as the generating argument will be apparent coordinates except for the earth where Ephemeris meridians, sidereal times, and longitudes will be generated. For the earth, apparent coordinates can be generated (not corrected for nutation) if U.T. is used instead of E.T.

Frequently in astronomy, intervals of mean solar days (ephemeris days) are greater than thirty. In order to overcome the inconvenience of month and year conversions, the concept of Julian days was introduced. Julian Days begin at Greenwich noon not midnight as do Civil Days. Hence 0.0 U.T. is 0.5 Julian Days. The fundamental epoch January $0^d\!.5$ 1900 E. T. = Julian Ephemeris Day 2415020.0.

Sec. 2.3 Establishment of Preliminary Coordinate References for Navigation Geodetic Exploration on Extraterrestrial Bodies.

In Sections 2.1 and 2.2, some concepts of terrestrial astronomy have been briefly reviewed. In this section, we apply these concepts to our particular problem - setting up a navigation ephemeris for the moon or another planet.

One assumption is made at the outset; it is assumed that for the body under consideration (i.e. the moon) the rotational motion is not well known enough to be able to make a distinction between ephemeris time intervals and rotationally defined "universal time" cannot be made so that sidereal time, meridian, and other terms take on their conventional meanings.

Consider a body whose axis of rotation is instantaneously (or longer) pointed in a specific direction. Perpendicular to this axis is an equatorial plane. Relative to the ecliptic plane, this equatorial plane makes a certain angle i (called the inclination or obliquity) at the line of intersection (see Figure 1). If a right-hand rule is used to define the rotation axis, the smallest angle between the planes is called i and the direction of intersection where rotation carries a point on the body from the northern to southern ecliptic hemispheres is called the descending node of the equator.

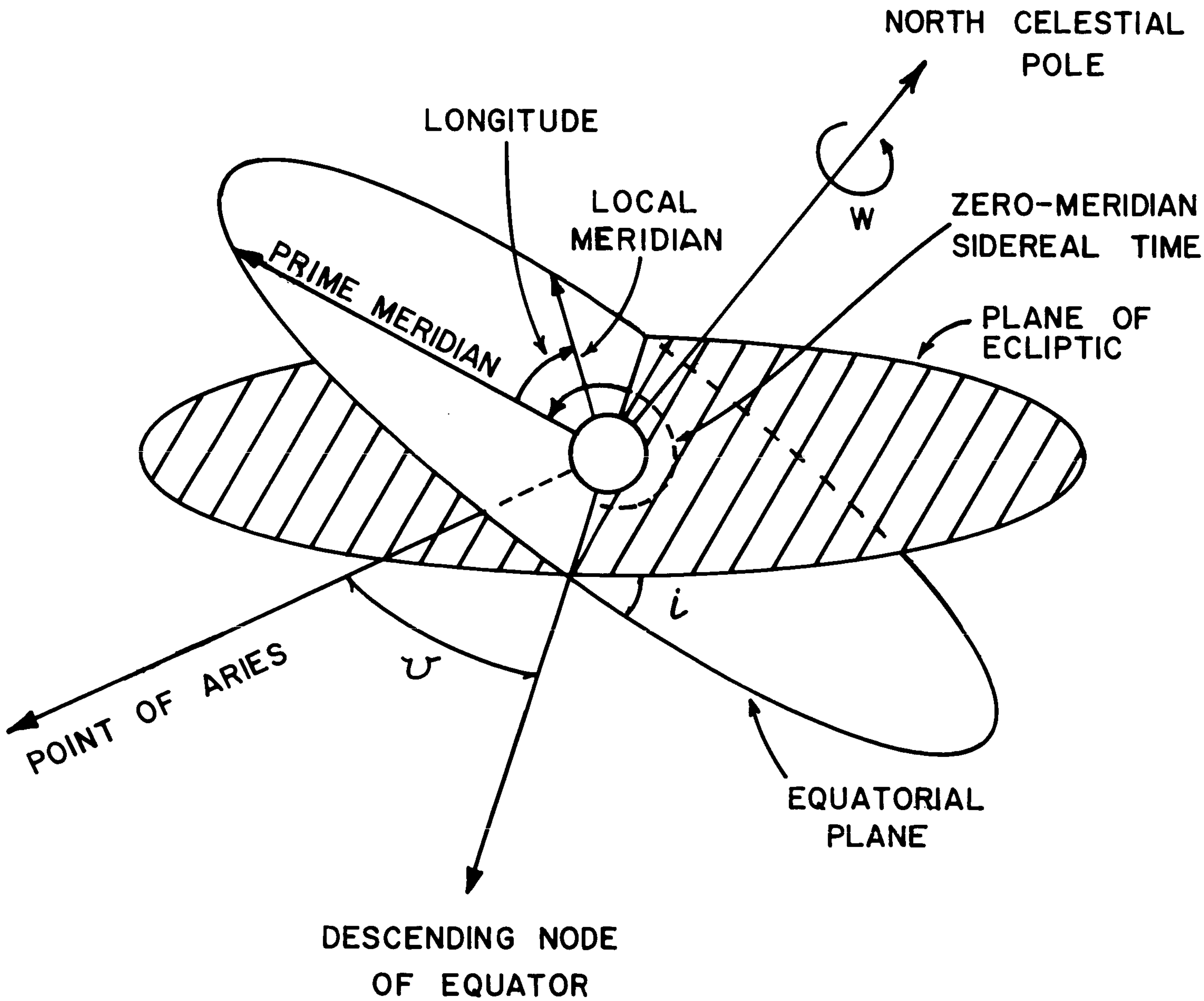
Coordinates relative to the ecliptic plane are spherical. Ecliptic (or celestial) longitude, λ is an angle measured counter-clockwise in the plane of the ecliptic from the point of Aries; ecliptic (or celestial) latitude is an angle β measured perpendicular to the ecliptic plane. The descending node is at a longitude of ☋.

Coordinates relative to the equatorial plane are also spherical. Right ascension (α or R.A.) is measured (from the descending node) counter-clockwise (right-hand rule) in the equatorial plane. Declination δ is measured perpendicular to the equator. Declination is expressed in angular units, but right ascension, for convenience, is divided into time units; 360 degrees equal 24 hours; 60 minutes equal an hour; 60 seconds equal a minute. (One hour of R. A. - 15° of R.A.). The angle (in time units) between the prime meridian is called the zero-meridian sidereal time. The difference in angle between the local meridian and the prime meridian is called longitude. Longitude is positive in a clockwise sense. The north celestial pole is the direction given by a right-hand rule. If ☋ and i are

FIGURE 1

GENERALIZED EQUATORIAL REFERENCE SYSTEM

note: FOR THE EARTH: $\upsilon = 0$, $i = \epsilon$ = obliquity of ecliptic



known it is possible to convert λ, β into α, δ by a trigonometric transformation.

$$\cos\delta\cos\alpha = \cos\beta\cos(\lambda - \text{☋})$$
$$\cos\delta\sin\alpha = \cos\beta\cos(\lambda - \text{☋})$$
$$\sin\delta = \cos\beta\sin(\lambda - \text{☋})\sin i + \sin\beta\cos i$$

The conversion from stationocentric, geocentric, or selenocentric coordinates (α, δ) to topocentric coordinates (altitude, azimuth) is also straight-forward. The various relationships are shown in Figure 2.

Consider a "celestial" hemisphere positioned over the equatorial plane. The radius vector to a "star" intersects the celestial sphere in a "subpoint." The subpoint makes an angle δ (= Declination) with the equatorial plane and an angle L.H.A. (= Local Hour Angle) with the local meridian of an observer. The prime meridian is located at an angle "LONGITUDE" from the local meridian. The local zenith is on the local meridian and is in the same plane as the north celestial pole. The zenith makes an angle "LATITUDE" with the equatorial plane. The relation between local sidereal time and zero-meridian sidereal time is also shown. The plane perpendicular to the zenith is called the horizon. A plane through the observer intersecting the horizon in an east-west line and parallel to the equatorial plane is called the observer's (or local) equatorial plane and is also shown in Figure 2. Concentric with the observer is the local celestial sphere. The local equatorial plane intersects the local celestial sphere in the celestial equator as shown in Figure 3. The descending node ☋ of the equator is confined to the equatorial plane. The angular distance from ☋ to the local meridian is ST, the local sidereal time. The angle ALT is the altitude of a star relative to the spherical horizon. The altitude of the north celestial pole is the spherical LATITUDE. The spherical (or local) zenith is the perpendicular to the horizon (local). The geoid zenith is defined as the normal

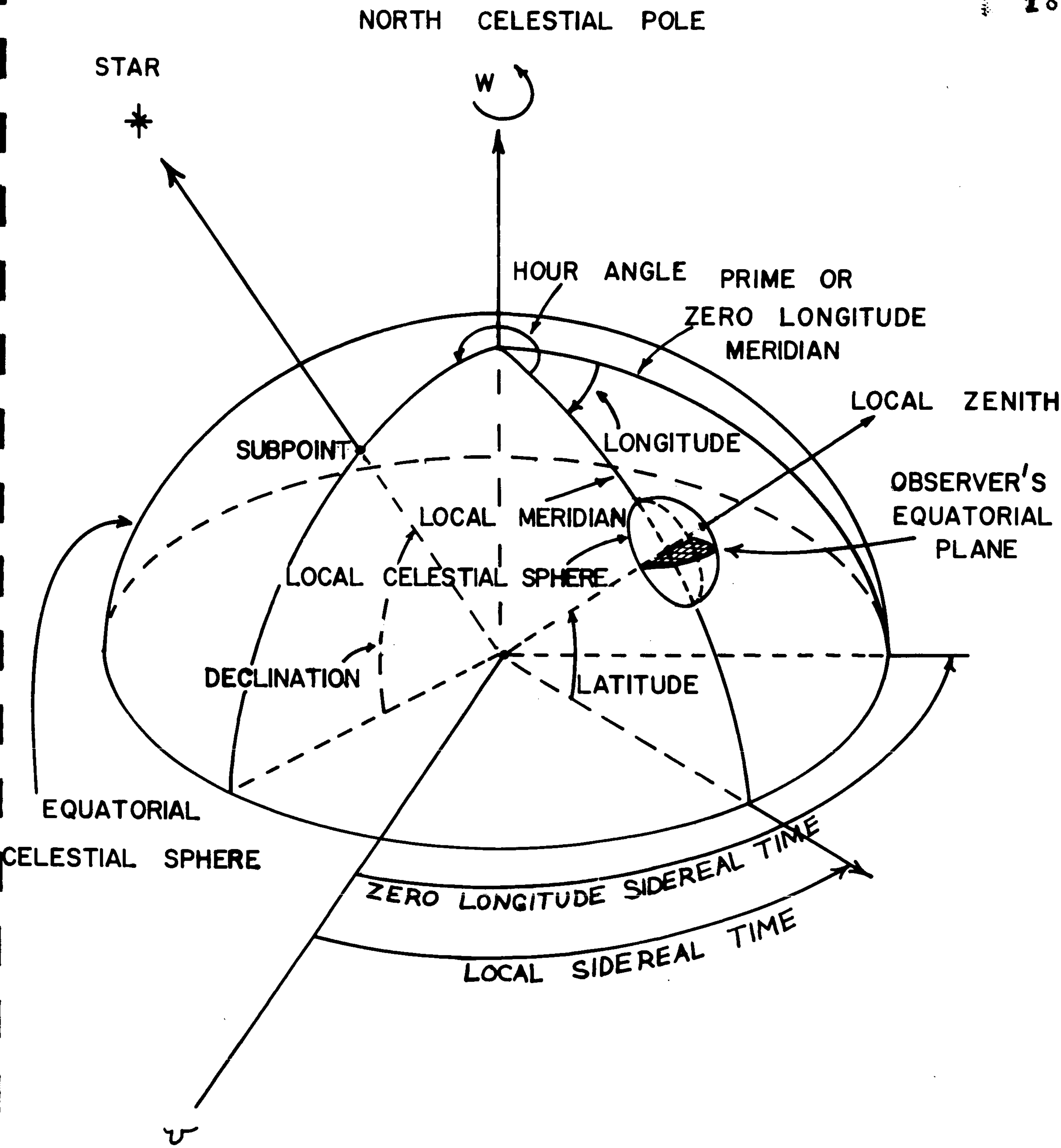
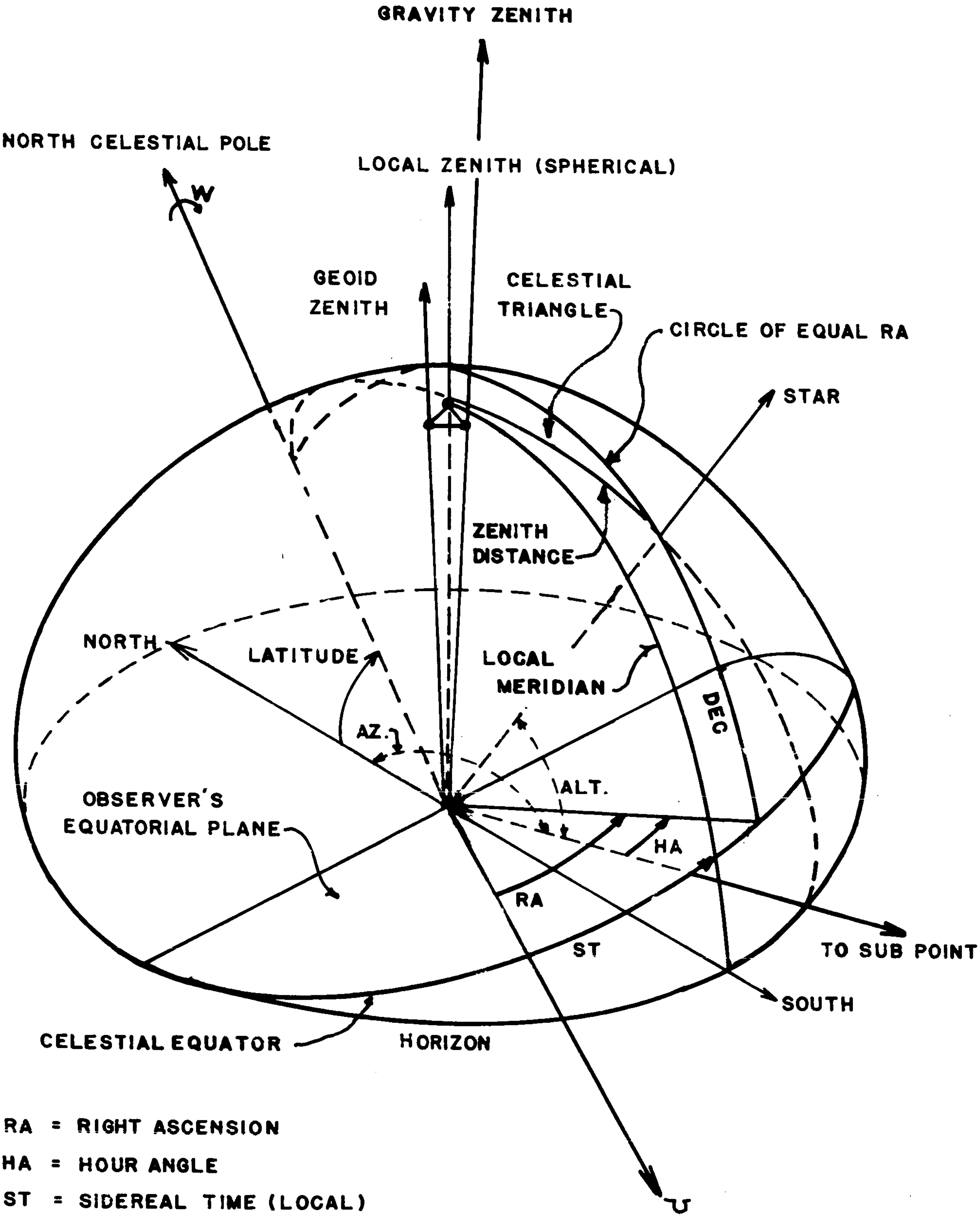


FIGURE 2. GEOCENTRIC AND TOPOCENTRIC POSITIONS



RA = RIGHT ASCENSION

HA = HOUR ANGLE

ST = SIDEREAL TIME (LOCAL)

DEC = DECLINATION

FIGURE 3- LOCAL REFERENCE SYSTEM

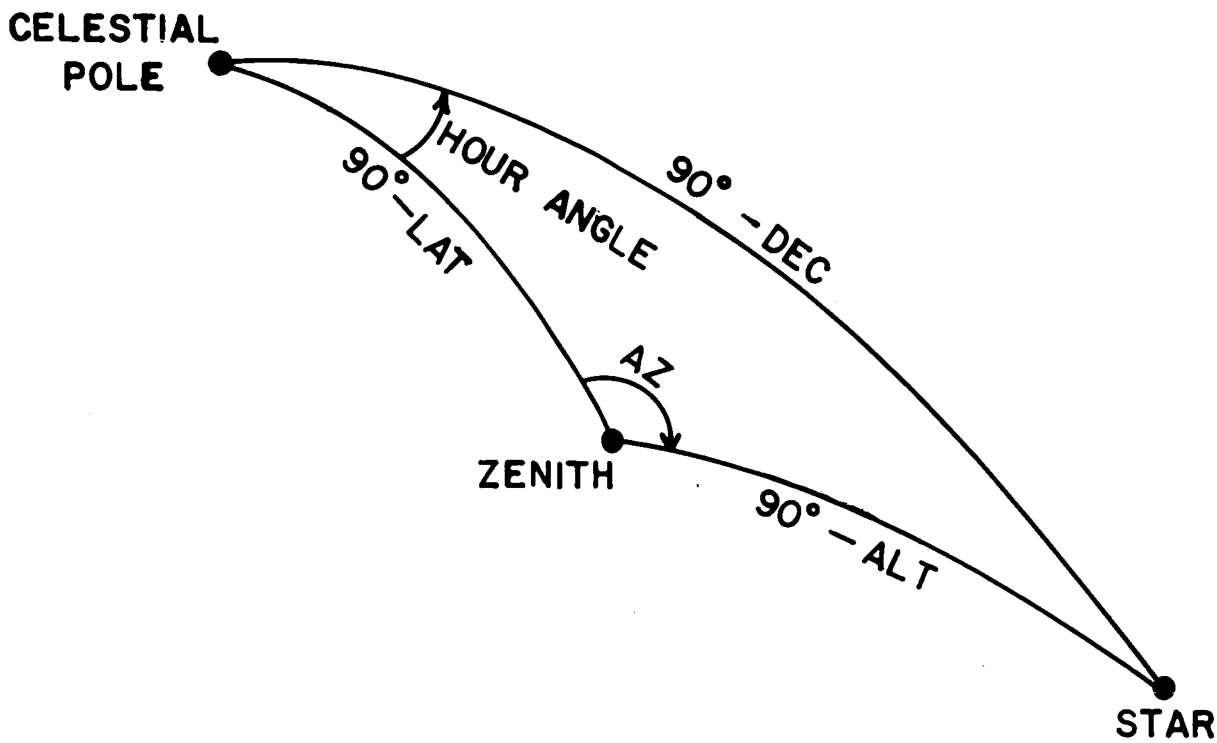
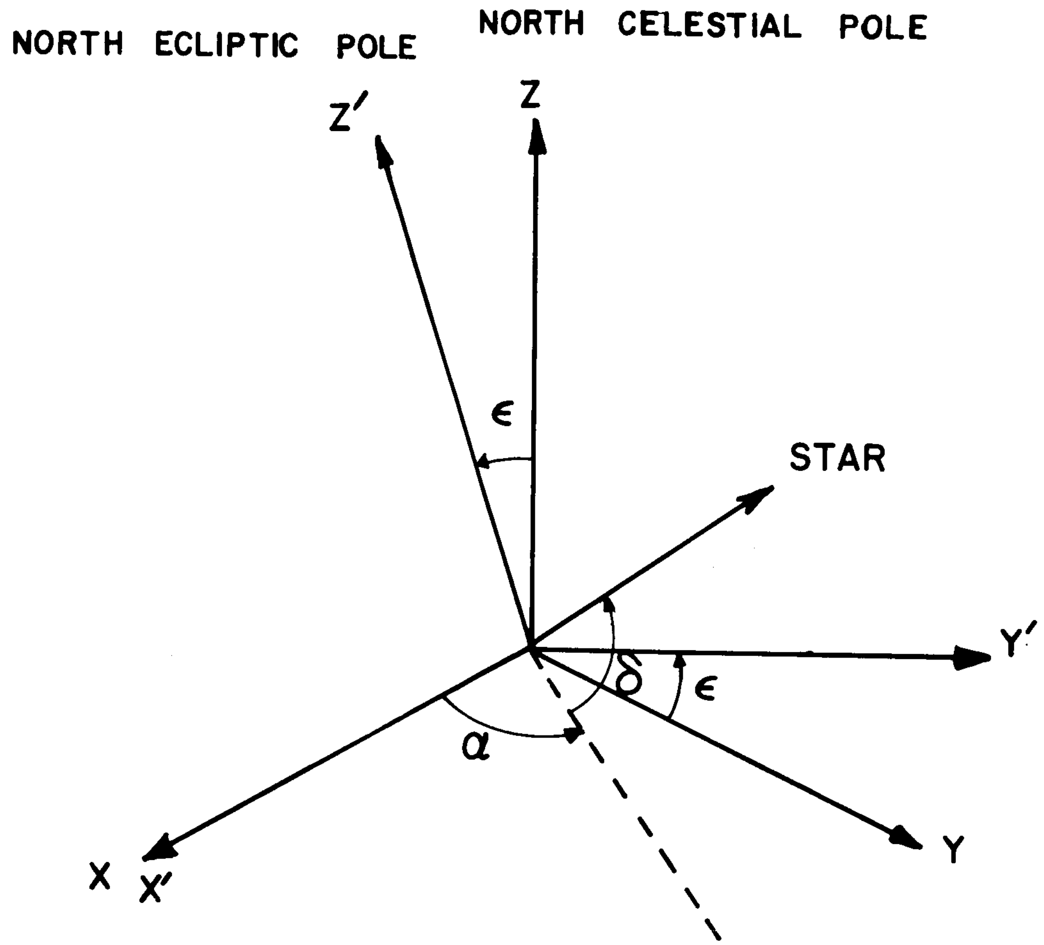


FIGURE 4 THE CELESTIAL TRIANGLE



XYZ SYSTEM EQUATORIAL

X′Y Z SYSTEM ECLIPTIC

α = Right Ascension

δ = Declination

ϵ = Obliquity of ecliptic

FIGURE 5 RELATION BETWEEN ECLIPTIC AND EQUATORIAL SYSTEMS to the tangent to the "geoid" at the position of the observer. The gravity zenith is the zenith which goes through the observer, but is parallel to the gradient of the gravitational field. The three zeniths coincide only if the "geoid" and the gravity field have spherical symmetry and the center-of-mass and center-of-figure coincide. The zenith distance is the angle between the star and the spherical zenith as seen by the observer. The topocentric right ascension is the angle, measured in the equatorial plane, from ☋ to the perpendicular (through the star) to the equatorial plane (RA). The local hour angle (HA) is (RA - ST). The declination (DEC) is the angle between the star and the equatorial plane. The azimuth of the star is the angle (AZ), measured clockwise from "north" to the direction of the subpoint of the star. The conversion between the RA/DEC system and the ALT/AZ system is accomplished using the so-called "celestial triangle" as shown in Figure 4. Using the triangle the "law of sines" and "law of cosines" the transformation equations are:

$$\begin{aligned}
\cos \mathrm{ALT} \sin \mathrm{AZ} &= -\cos \mathrm{DEC} \cos \mathrm{HA} \\
\cos \mathrm{ALT} \cos \mathrm{AZ} &= \sin \mathrm{DEC} \cos \mathrm{LAT} - \cos \mathrm{DEC} \cos \mathrm{HA} \sin \mathrm{LAT} \\
\sin \mathrm{ALT} &= \sin \mathrm{DEC} \sin \mathrm{LAT} + \cos \mathrm{DEC} \cos \mathrm{HA} \cos \mathrm{LAT} \\
\mathrm{HA} &= \mathrm{RA} - \mathrm{ST}.
\end{aligned}$$

It can be seen that the earth equatorial system is obtained if the angle between the descending node and the point of Aries is zero (☋ = 0). The conversion is simply a rotation relative to the X'Y'Z' ecliptic coordinates of the XYZ equatorial system through the angle ☋ (see Figure 5). Likewise, the altitude-azimuth conversion is a rotation through an angle (90° - LAT) in the meridian plane.

The ecliptic coordinates of an object are the same regardless of the equatorial reference used. In the LANS program-reductions, the conversion is made from X, Y, Z to X', Y', Z' (Figure 5) to λ, β and distance. Then, these are converted using spherical trigonometry to an α, δ, distance system. The α, δ, thus obtained, however, do not <u>necessarily</u> correspond to the earth α, δ system (see Figure 6). This double conversion has the advantage that only the precession of the generalized system has to be computed. The normal procedure (Astronomical Ephemeris) is to convert directly from earth α, δ to the generalized α, δ without finding λ and β. But although this does involve fewer initial calculations and is more adaptable to logarithmic computation, it has no advantage for LANS or similar computations where electronic computers are used. It also requires the precessional computations for the earth as well as for the other body.

Before the coordinates λ, β can be found, a transfer of origin is necessary. The available planetary theory tables (ephemeridies) are <u>heliocentric</u>, <u>rectangular</u> (earth) equatorial coordinates, epoch 1950.0. The positions of the sun are 1950.0 <u>geocentric</u> equatorial coordinates, which if negated give heliocentric coordinates of the earth. The positions of the moon are also 1950.0 <u>geocentric</u> (earth) equatorial coordinates. Since all of these coordinates are equatorial, rectangular coordinates, a change or origin can be accomplished rather easily. The vector relations are shown in Figure 7.

If $\bar{R}$ = geocentric vector of sun, $\bar{p}$ = seleocentric vector of sun, $\bar{r}$ = geocentric vector of moon, r_p'' = selenocentric vector of planet, $\bar{r}_p$ = geocentric vector of planet,

$\bar{r}_p'$ = heliocentric vector of planet,

$(-\bar{p})$ = heliocentric vector of moon (or station),

$\bar{r}_{cm}$ = heliocentric vector of center of mass of solar system,

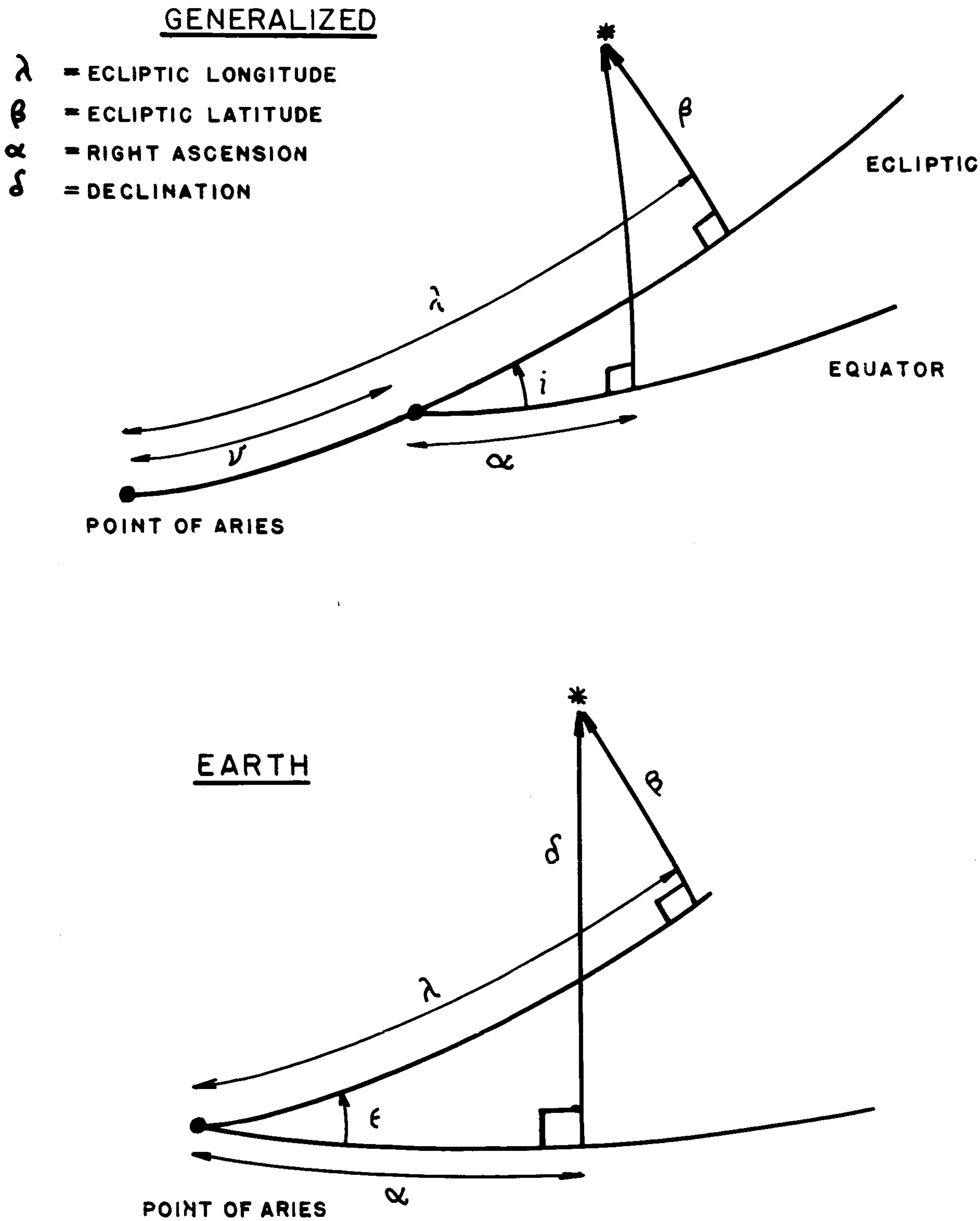
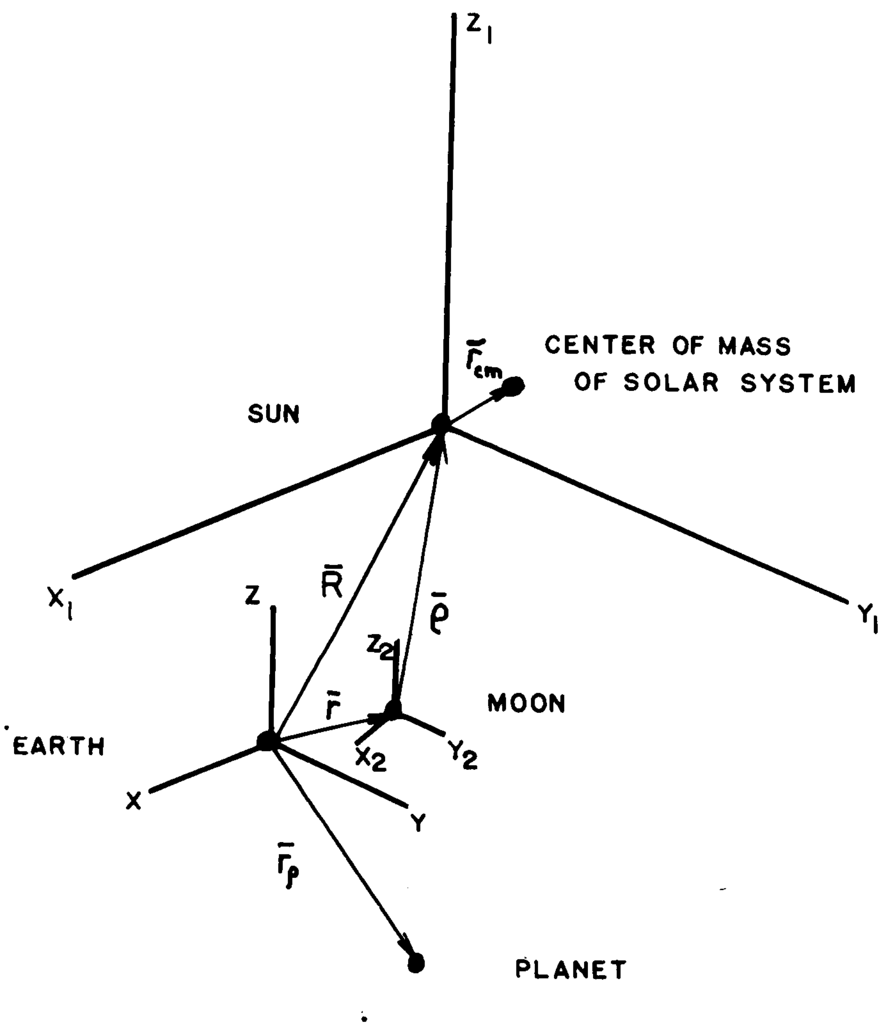


FIGURE 6 - SPHERICAL FIGURES BETWEEN EQUATORIAL AND ECLIPTIC ANGLES



X_1, Y_1, Z_1 HELIOCENTRIC RECTANGULAR COORDINATES

X, Y, Z GEOCENTRIC RECTANGULAR COORDINATES

X_2, Y_2, Z_2 SELENOCENTRIC RECTANGULAR COORDINATES

$\bar{R}, \bar{r}, \bar{\rho}, \bar{r}_p, \bar{r}_{cm}$ ARE RADIUS VECTORS

FIGURE 7- RELATIONS BETWEEN HELIOCENTRIC, GEOCENTRIC, AND SELENOCENTRIC RECTANGULAR EQUATORIAL COORDINATES

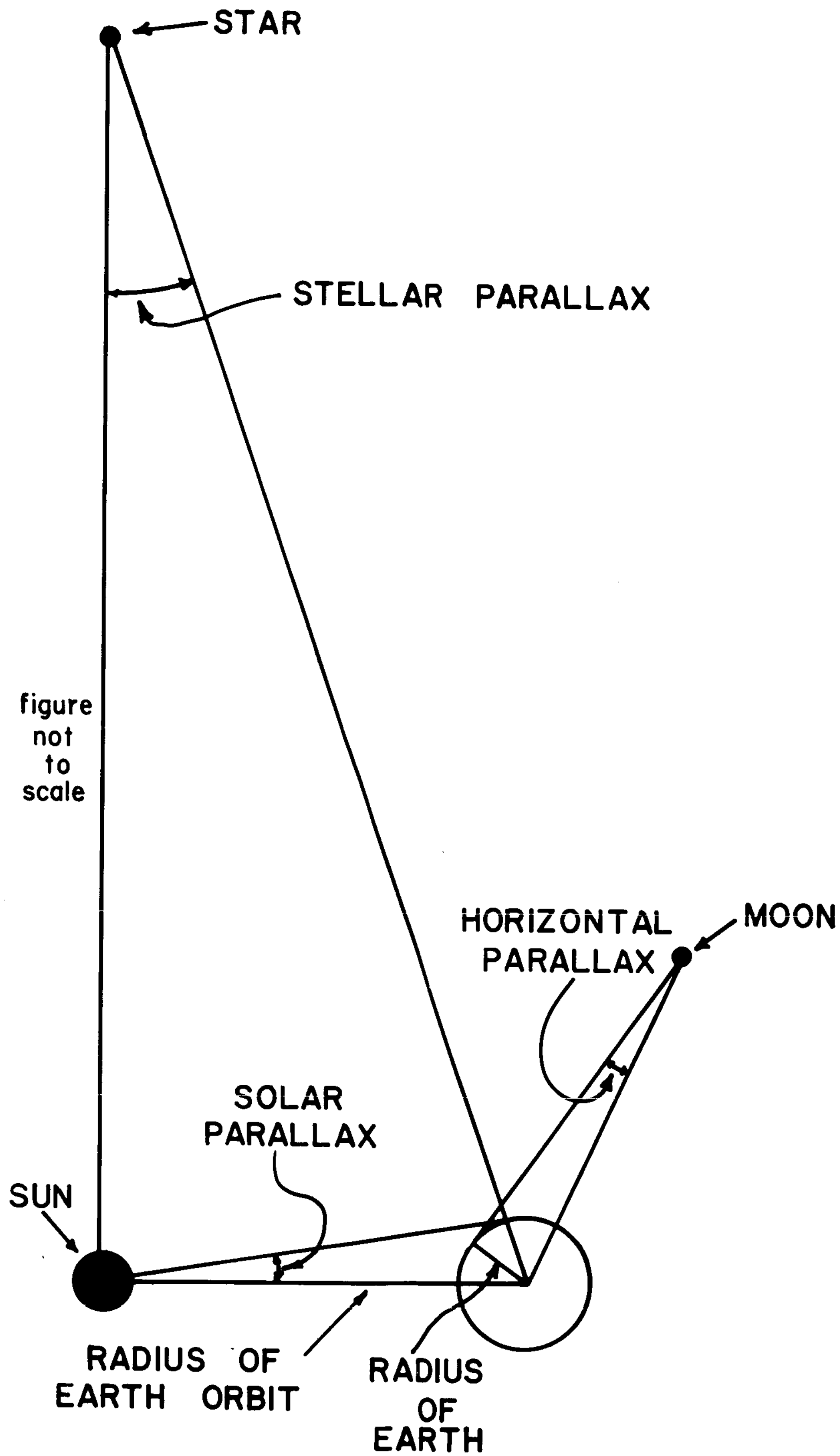


FIGURE-8 PARALLAXES

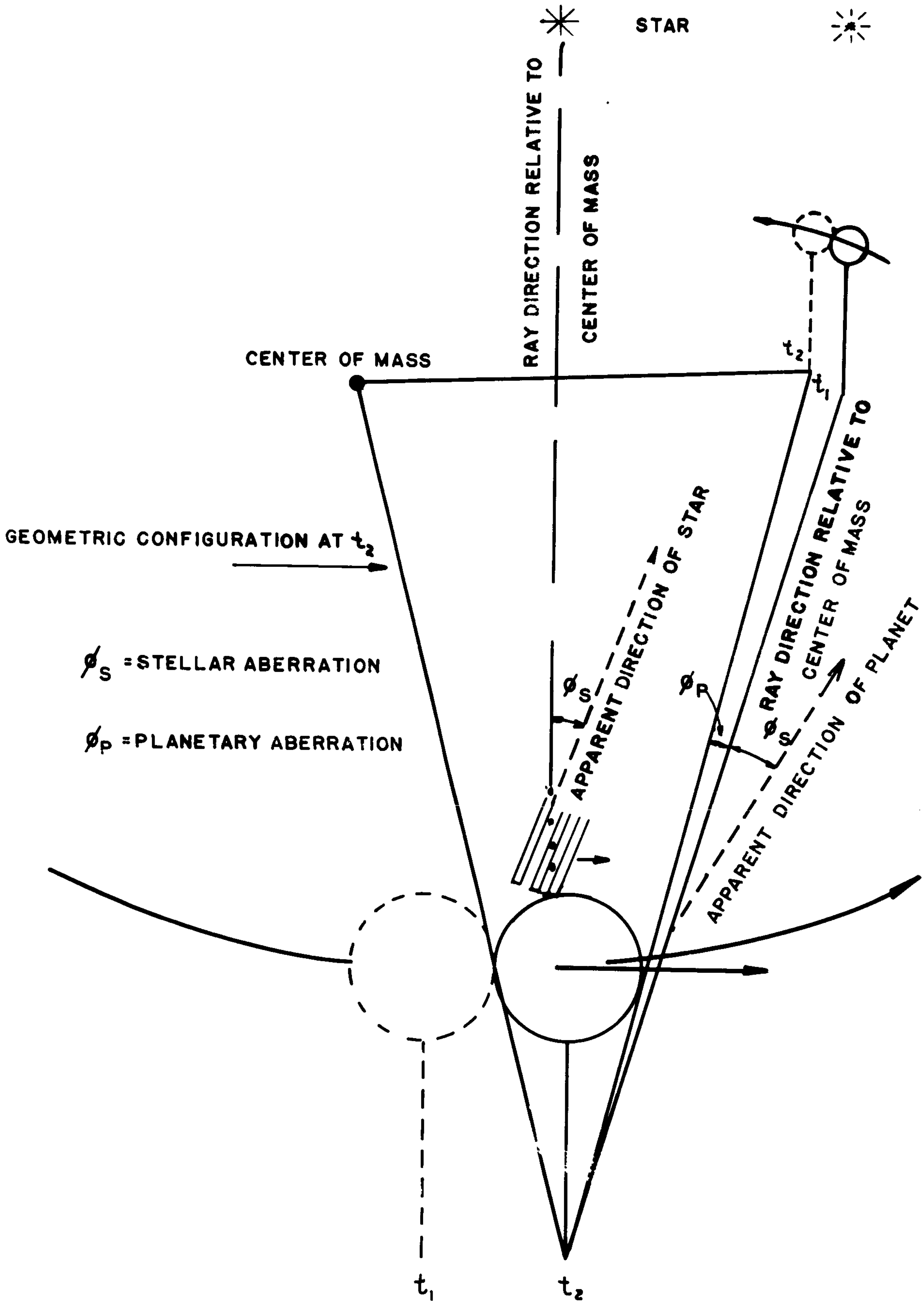


FIGURE 9 - ABERRATIONS

thus a reduction to a center-of-mass system and to a selenocentric or stationocentric system is expressed by the vector relations:

Station

$$\bar{r} + \bar{P} = \bar{R} \quad .....(1)$$

$$\bar{r}_p = \bar{R} = \bar{r}'_p \quad .....(2)$$

$$\bar{r}'_p + \bar{p} = \bar{r}''_p \quad .....(3)$$

$$\bar{r} - \bar{R} = (-\bar{p}) \quad .....(4)$$

Center of Mass

$$(-\bar{p}) - \bar{r}_{cm} = (-\bar{p})_{cm} \quad .....(5)$$

$$\bar{R} + \bar{r}_{cm} = \bar{R}_{cm} \quad .....(6)$$

$$r'_p - \bar{r}_{cm} = (\bar{r}'_p)_{cm} \quad .....(7)$$

These transformations require no parallax corrections. However, the positions of the stars do require a correction for stellar parallax if the "base-line" orbit size is changed. For the moon, the stellar parallax corrections are nearly negligible except for the nearest stars. However, the generalized concepts used in LANS require that it be included if the heliocentric distance is much different from unity. Figure 8 shows the three main parallaxes of concern in astronomy and the base-lines which are used to measure them. In LANS, a <u>lunar</u> horizontal parallax correction is applied, if necessary, before the final <u>topocentric</u> coordinate conversion. The base-line in this case is the station center-of-mass radius vector.

Figure 9 illustrates the aberration of light as discussed in some detail in Sec. 2.1. Consider two bodies moving relative to an inertial frame origin (center-of-mass). From a classical standpoint, a star is considered fixed relative to the center-of-mass. Then as a photon travels in a straight path relative to the center-of-mass, a telescope on one of the moving bodies has to be tilted so that it can "catch" the photon. Unless the telescope <u>is</u>

tilted, the photon "collides" with the side of the tube. Since all photons are affected, the stars all appear "tilted," hence stellar aberration. It results from motion of the observer. The planetary aberration component ϕ_s is the difference in ray angle due to the object motion. If ϕ_p is the stellar aberration component, then the total planetary aberration ϕ is

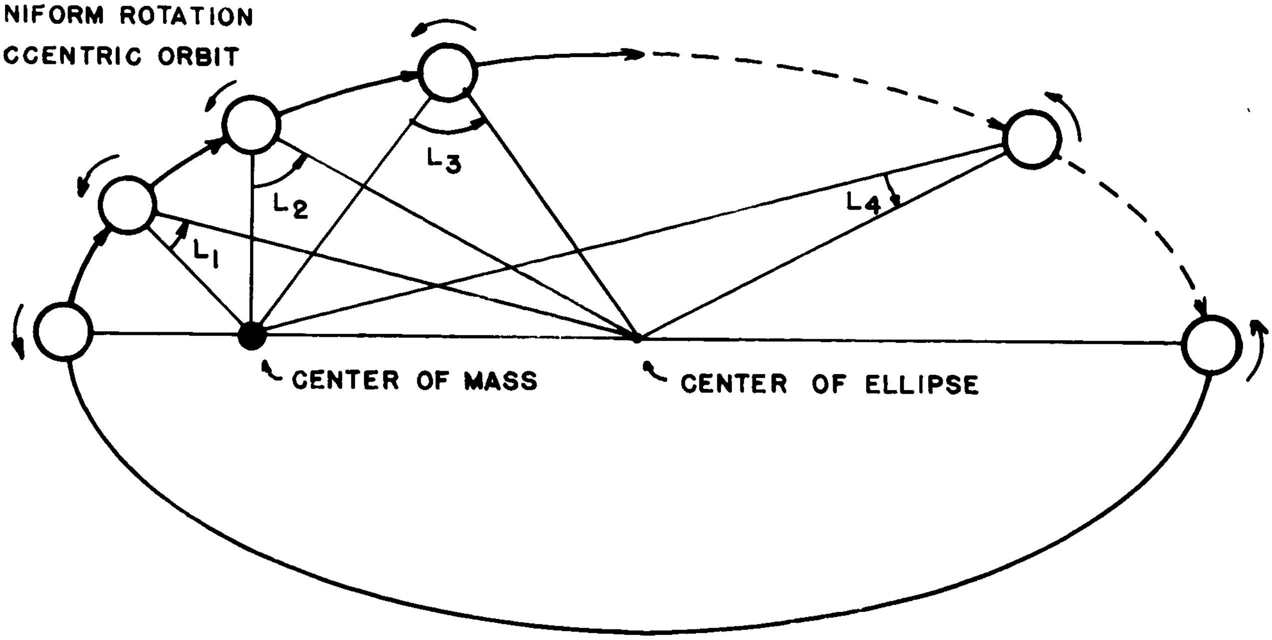
$$\phi = \phi_s + \phi_p \simeq \frac{\text{(relative tangential velocity)}}{\text{(velocity of light)}}$$

Secular aberration is not shown. The stellar aberration is more correctly termed annual aberration, if the orbital motion of the observer is assumed circular. In LANS, stellar aberration is computed using numerical diffentiation of the observer's coordinates and planetary aberration is computed using interpolation back to an epoch $(t_o - \tau)$ where τ is the light-time.

The rotation rate of the observer is used to compute the diurnal aberration, if this correction is necessary. For the moon, this correction is negligible, but the capability is included for generality.

In Sec. 2.1, it was mentioned that the combined effect of precession and nutation was called the physical libration. This is not to be confused with the optical librations which are demonstrated in Figure 10.

Consider for the moment that the body (moon) rotates at exactly a uniform rate. A prime meridian then sweeps out equal areas and equal angles in equal times. Suppose, however, that the body is in an elliptical orbit about some center-of-mass. While the body rotation sweeps out equal angles, the Keplerian motion certainly does not, but only sweeps out equal areas. Of course, if the orbit were circular, the rotation and revolution would be exactly synchronized. The difference in angle between the radius to the



L_1, L_2, L_3, L_4 ARE REPRESENTATIVE LONGITUDINAL LIBRATIONS

L_5, L_6 ARE REPRESENTATIVE LATITUDINAL LIBRATIONS

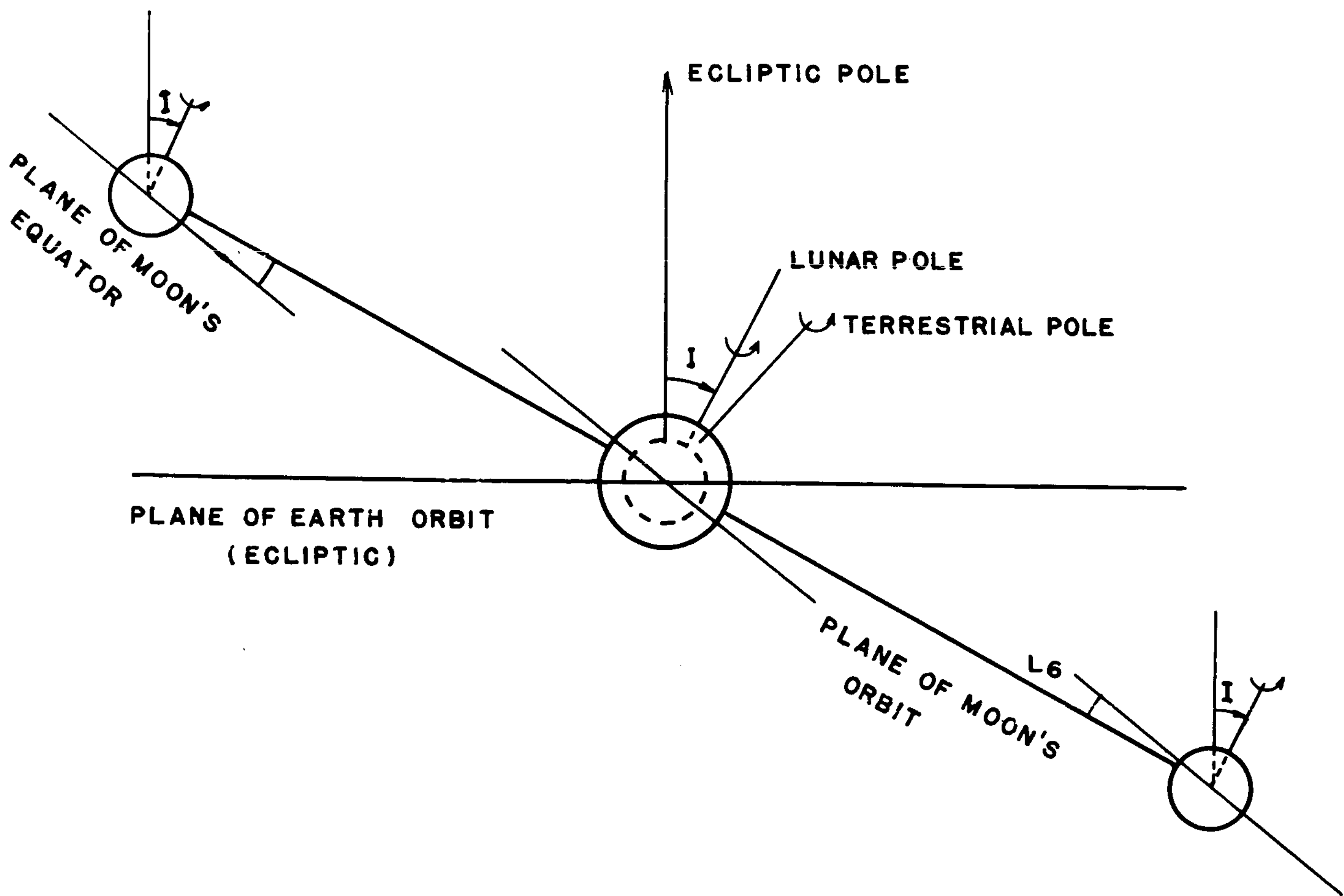


FIGURE 10- OPTICAL LIBRATIONS

center-of-mass and the radius to the center of the ellipse is called the longitudinal libration (optical). If there are small variations of rotation rate, these show up as physical longitudinal libration.

Suppose now that the body rotation axis is not parallel either to the terrestrial pole or to the perpendicular to the orbital plane. Thus the sub-earth point will be alternately above and below the lunar equator. These are called (optical) Latitudinal Librations. The physical latitudinal librations are principally due to nutation of the body axis. It should be noted that because of precession, the long-term oscillations in longitude are not strictly periodic.

The lunar physical librations have recently been studied by Eckhardt (1965) at the Air Force Cambridge Research Laboratories. Dr. Eckhardt kindly provided us with a grid of libration constants over a $\pm 3\sigma$ range around the β, γ best values now available, especially for use in LANS. The current version of LANS uses libration constants which are considered to be the best currently available. Although only 22 coefficients are known, there is room for 45. This was done so that a single or a few constants could be changed or added without disturbing the program itself. If the "best" values for the moments-of-inertia are incorrect, an entirely new set of constants may be generated using the auxiliary two-dimensional interpolation program. The storage requirements for such a program internal to LANS is prohibitive for the general case.

The Eckhardt solutions are for the forced librations only. The solutions of the "free-oscillations" have not been included, since they can only be evaluated empirically. Likewise, long-period forcing terms are not included although they probably do exist. Not enough empirical evidence for free-librations or long-period terms is available. If and when coefficients are available, it will be a simple matter to include them in LANS. ALL the necessary auxiliary angular arguments are already available internally. Short-term corrections to inaccuracies in the libration constants may be obtained from observation using the empirical data reduction program in LANS. The corrections, however, do not indicate which periodicities are present and which are not. Such a study to find the long-term and free librations has been proposed as a contract extension of this investigation, but has not yet been taken up in detail.

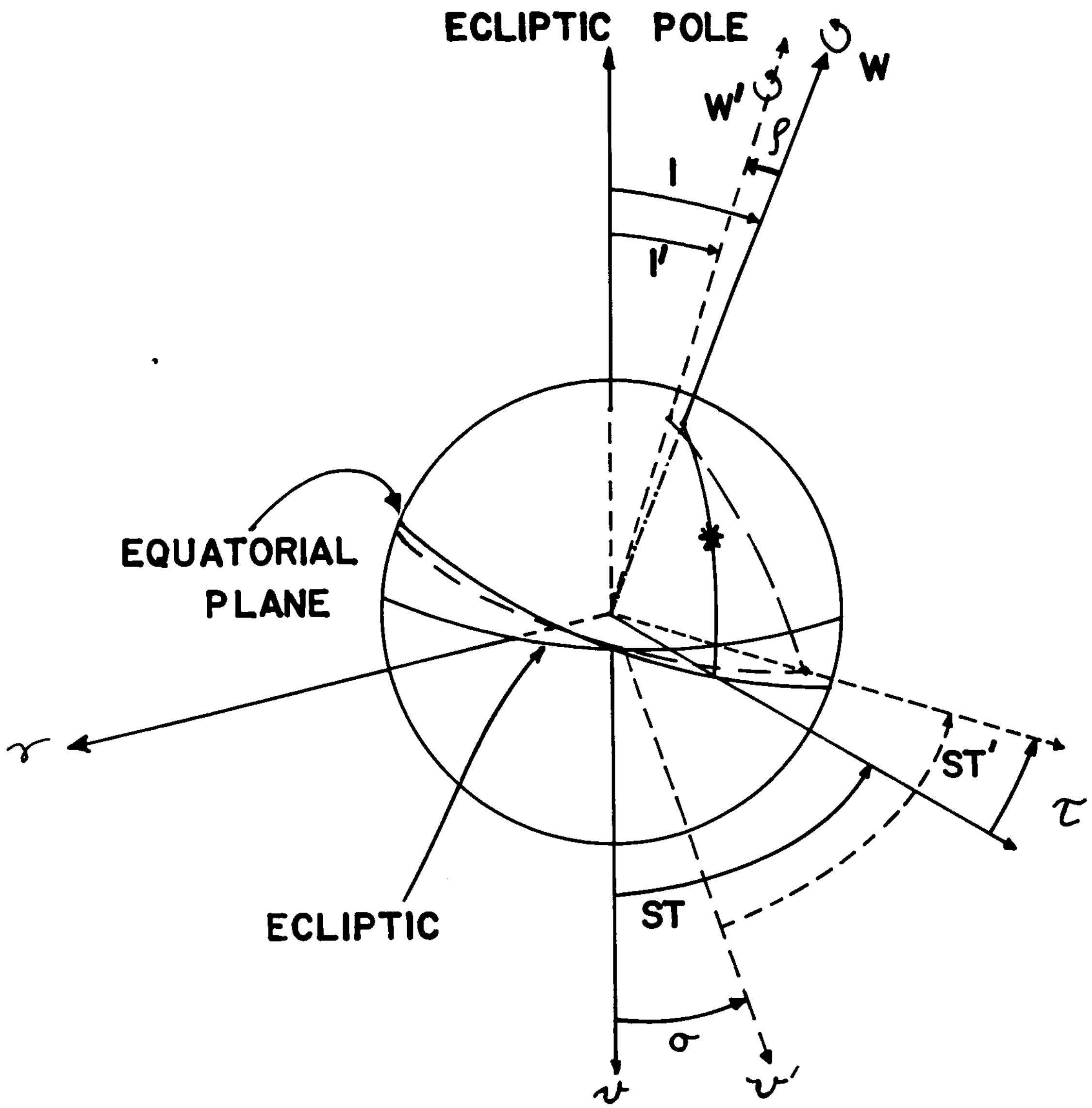
Figure 11 illustrates the lunar physical librations. These librations are computed under the following assumptions:

a a) The luanr rotation rate is uniform and constant, to the first order. The lunar prime meridian has a constant angle with the <u>mean</u> longitude of the moon.

b) The lunar precession has the same period as the precession of the <u>line of nodes of the orbit</u> and, in fact, the equatorial line of nodes coincides with the orbital line of nodes.

c) The <u>mean</u> axial inclination of the moon is constant at least to first order.

The above constitute <u>Cassini's Laws of Rotation</u>. These are only true to the first order. In a physical sense then, the (physical) librations represent perturbations to $\bar{W}$, the mean rotation rate, to $\bar{I}$, the mean axial inclination, and to $\mho$, the mean longitude of the descending node of the equator. Since $\bar{\mho}$ has a known secular part, the precession, it is usually removed before the physical librations are solved for $\mho$. Thus if W', I',



ORIGINAL SYSTEM [W = ANGULAR ROTATION RATE = W'] NEW SYSTEM

I = AXIAL INCLINATION = I'

υ = POSITION OF NODE = υ'

♈ = POINT OF ARIES

σ = LONGITUDINAL COMPONENT

ρ = LATITUDINAL COMPONENT (NUTATION)

τ = ROTATIONAL COMPONENT

(THE SECULAR PART OF σ, IF IT EXISTS, IS CALLED PRECESSION AND IS USUALLY CONSIDERED SEPARATELY)

FIGURE II — PHYSICAL LIBRATIONS

and $\mho'$ are the instantaneous values, these are assumed only to be a small amount different from $\overline{W}$, $\overline{I}$, $\overline{\mho}$. Thus

$$\begin{aligned} W' &= \overline{W} + \Delta W \\ I' &= \overline{I} + \Delta i \\ \mho' &= \overline{\mho} + \Delta \mho \end{aligned}$$

These perturbations are not convenient for computation of coordinates, so another convention is used.

σ = longitudinal libration
ρ = latitudinal libration
τ = rotational libration

The relationships of the small angles are shown in the diagram (Figure 11) and are covered in detail by Eckhardt (1965).

Sec. 3 SPECIFICATION OF THE LUNAR PROBLEM

In the proceeding sections, we have discussed the problem of astronomical ephemeris data from a generalized standpoint. In this section we concentrate on the problem of celestial navigation from the lunar surface.

The calculation of apparent selenocentric object coordinates with complete aberrational and parallactic corrections from heliocentric or geocentric data is straight forward, although lengthy. The appropriate transformations are performed in the basic program of LANS in accordance with the recommendations for ephemeris calculation as outlined in the Explanatory Supplement to the Astronomical Ephemeris (1960). Likewise, the topocentric coordinates and corrections have been computed with notation and convention similar to those used on the earth. A conscious effort has been made to maintain as close analogy with the problem of celestial navigation on earth as was possible. Some differences do remain which are discussed in this chapter.

The lack of a permanent or appreciable atmosphere on the moon eliminates refraction as a factor in the lunar navigation so this was not considered. The Explanatory Supplement has a short section (2E), but includes a number of more important astronomical references.

The Astronomical Ephemeris for the earth contains the following data for the mean lunar equator, (quoted from the Explanatory Supplement):

a) i = the inclination of the mean equator of the moon to the true equator of the earth.

b) Δ = the arc of the mean equator of the moon from its ascending node on the true equator of the earth to its ascending node on the ecliptic of date.

c) Ω' = the arc of the true equator of the earth from the true equinox of date to the ascending node of the mean equator of the moon. For the purposes of a precise lunar ephemeris these quantities are not satisfactory for a number of reasons.

1) These are derived assuming Hayn's approximate mean inclination of the lunar equator to the ecliptic of $I = 1°32'$

2) These do not include nutation of the lunar pole.

3) They refer to the true equator of the earth not the 1950.0 equator.

The "Ephemeris for Physical Observations" of the Astronomical Ephemeris is somewhat better, but also has shortcomings.

Hayn's values for the mean lunar equator and the physical librations constants were used to derive σ, τ, ρ and then calculate:

a) Selenographic longitude and latitude of the earth.

b) Selenographic position of the sun.

The calculations are discussed in the Explanatory Supplement pp. 316-326. While these methods are reasonable satosfactory for an earth ephemeris where only $0\overset{\circ}{.}001$ accuracy is needed, the Hayn (1907) values adopted are probably not the most accurate. In addition, a number of approximations and auxiliary quantities are introduced which are not required for electronic data processing. We have therefore reformulated the problem.

Solutions for lunar nutation have been recently derived by Eckhardt (1965) using direct numerical integration of the equations of motion. Hayn (1907), however, did not have a modern high-speed computer and so his investigations involve a number of approximations. Koziel (1962), Jeffreys (1961), Watts (1955), and Goudas (1966)have made some more recent determinations of parameters directly or indirectly influencing the libration coefficients. The review article by Koziel (1962) should be consulted for a discussion of the work previous to 1958. The notation used by Hayn has been retained. The arguments used for the computation lunar librations are the same as those used to compute the <u>nutation</u> of the earth's axis, namely 1, 1', F, D, and Ω.

$l = L - \tilde{\omega}$ = mean anomaly of moon

$l' = L' - \tilde{\omega}'$ = mean anomaly of sun

$F = L - \Omega$ = mean nodal distance of moon

$D = L - L'$ = mean elongation of moon from sun

L = mean longitude of moon, $\tilde{\omega}$ = argument of perigee

L' = mean longitude of sun, $\tilde{\omega}$ = argument of perihelion

Ω = ascending node of the moon.

Eckhardt (1965) gives expansions of σ, ρ, τ, and values of I for various values of moment of inertia parameters. He has kindly supplied tables of libration constants especially for LANS. An interpolation scheme allows computation of the coefficients. These with subsequent corrections, if necessary, can be entered into LANS from outside.

The moment of inertia parameters used for coefficient interpolation are α, β and γ. But

$$\gamma = -\beta(1 - f) \qquad f = -\alpha/\beta$$

so only two values β, γ or β, f need be specified. The parameters α, β, γ are defined in terms of the principal moments of inertia A, B, C by

$$\alpha = (B - C)/A \quad \beta = \frac{C - A}{B} \quad \gamma = \frac{B - A}{C}$$

The tables provided by Eckhardt are in terms of β and f over the ranges

$$0.59 < f < 0.65$$

$$0.000\,60 < \beta < 0.00066$$

Now the best value of f Jeffreys(1961) is $f = 0.639 \pm 0.014$; that for γ is $+0.000\ 2274 \pm 0.000\ 0088$ which results in a β of 0.000 637. Thus, the values of f and β are still uncertain and over much larger ranges than the quoted probable errors seem to imply. There is considerable evidence which indicates however, that β and f are within the ranges stated above. For initial table generation tests $\beta = 0.00064$ and $f = 0.64$ where chosen.

When the Lunar Orbiter data analysis is complete very accurate values of β and f should be available and the libration coefficients may be determined with more confidence (Michael and Tolson, 1965). At present this is the best that can be obtained. We feel that the values used are more realistic than those used in the Russian ephemeris (see section 1) computed by Yakoukin, Demenko, Miz' in 1964, but more data is needed to establish this. After the libration constants are specified it is possible to obtain the apparent axial coordinates as seen from the center of the moon. Thus apparent lunar right ascension, declination can be found for any object whose geometric (gravitational) ephemeris is available. As a consequence of the correction for librations, it is possible to find lunar <u>apparent</u> sidereal time, unlike terrestrial time, is relatively non-uniform even over relatively short intervals. One could, as in the case of the Russian ephemeris, define a uniform sidereal time for the moon, but we found this of no practical value since ephemeris is a uniform time which is well-known and presumably will be available to users of lunar navigation tables. Likewise, no uniform lunar solar time is defined or used.

If the sun is treated as a planet, the zero longitude sidereal time and hour angle of the sun are obtained in an intermediate step in the LANS topocentric conversion procedure. They may be outputed, if desired, by setting an option. These are apparent times and are non-uniform. Once zero-longitude sidereal time, right ascension, and declination at the center of the moon have been found, the topocentric conversion can be made.

The topocentric conversions are really a geodetic astronomy problem. Because of the rather high degree of symmetry which the gravitational field and "geoid" of the earth possess, topocentric corrections accurate enough for most purposes assume that only the lines of longitude are not great circles. These are usually considered to be ellipses. It is also assumed that the center-of-mass and center-of-figure of the earth coincide.

In the case of the moon, there are several complications:

1) The center-of-mass and center-of-figure of the moon do <u>not</u> coincide.

2) The geoid and gravity field of the moon are too irregular to permit accurate description of topocentric corrections by assuming simple geometric shapes.

3) <u>Local</u> Deflections of the gravity vertical may be considerable, but no data is yet available on these.

Complications (1) and (2) are taken into account in LANS, but only estimates of (3) can be given at the present time. These will be discussed later

The center-of-mass is origin for astronomical selenocentric coordinates.

It is also the origin for selenocentric coordinates derived using the gravity field. The center-of-figure, however, is the coordinate origin for geodetic studies of the shape of the moon. It is also the origin for the normally defined spherical longitude and latitude system. If the two centers are not coincident corrections must be made.

In LANS, topocentric corrections are made relative to the center-of-figure system. Corrections for horizontal parallax are made first by a translation to the center-of-figure, then to the surface.

The lunar geoid and gravity field have been investigated using spherical harmonics by Goudas. In a recent review article (1966), he derives expressions for an eighth order geoid; then he derives, on the assumption of a homogeneous mass distribution, a fourth order spherical harmonic expansion for the lunar potential field. For a number of the coefficients, the values depend strongly on the data used to derive them. These represent the best data available at the time of compilation of LANS so they have been incorporated into the basic program. These functions are generated as functions of geodetic radius and spherical longitude and latitude (center-of-figure). However, only the fourth order expansion of the geoid and 4th order gravity field give reasonable values.

When Lunar Orbiter data becomes available, it is very likely that the lunar gravity potential will be generated as a function of the coordinates relative to the center-of-mass. If this is true, setting an option will make the conversion from center-of-figure to center-of-mass before generating the gravity parameters.

In the problem of celestial navigation, a number of methods of solution utilize altitude - azimuth reference systems. On the moon, there are three topocentric reference systems which might be useful. The spherical system uses the spherical radius vector at the given LONGITUDE/LATITUDE for the zenith reference. The surface system uses the normal to the geoid for the reference. The <u>gravity system</u> uses a "smoothed" gravity vector for the zenith reference.

In practice, none of these systems is available to an observer without some knowledge of local conditions. On the surface of the earth it is usually assumed that the <u>surface system</u> and the <u>gravity system</u> coincide. At sea, this is rigorously true since for a fluid, the local shape is determined by the gravity field. But on the moon there are no fluid seas. If there were any "dust" maria on the moon, these could be used in an analogous manner to terrestrial seas, but evidence now appears to indicate that the maria are definitely <u>not</u> "dust-bowls." The maria, however, do appear to be relatively "flat." Horizon references might be defined for these areas, as can be done in plains or deserts (to a limited extent) on the earth, but the precision will be lower than for a gravity reference. A surface reference system, however, can be used to determine relative slopes of terrain.

The spherical system is the standard astronomical reference. It is mathematically much simpler than the gravity or surface system, but it is not practical for accurate navigation. A <u>spherical</u> system referenced to the center-of-mass can be obtained upon option from LANS, but is not used for computation.

The most practical system available for navigation is one using a gravity reference. Although the gravity vector is more constant in direction than the normal vector to the "horizon," it is subject to local variations, which are called <u>deflections of the vertical</u>. (It should be noted that deflections of the vertical occur for "geoid" reference systems as well as for gravity ones).

For our purpose here, we define a deflection of the vertical as being made up of two components:

a) <u>general deflection</u> of the vertical is the angular difference between the spherical zenith and either the geoid zenith or the "smoothed" gravity zenith.

b) <u>local deflection</u> of the vertical is the "local" stochastic angular difference between the true local gravity and surface zeniths and their <u>computed</u> "smooth" values.

The <u>total</u> deflection is the general plus the local deflection and represents the deflection of the true zenith from the spherical one. For the moon, the general deflections can be rather sizeable, much larger than for the earth and are very non-linear. It can be shown also that the maximum local deflections can also be rather large (see later section).

In LANS, altitudes and azimuths of objects can be referenced to a number of different systems upon option.

A. SPHERICAL - CENTER-OF-FIGURE
B. SPHERICAL - CENTER-OF-MASS
C. TRIAXIAL ELLIPSOID - CENTER-OF-FIGURE
D. TRIAXIAL ELLIPSOID - CENTER-OF-MASS
E. GENERAL HARMONIC SURFACE - CENTER-OF-FIGURE
F. GRAVITATIONAL POTENTIAL FIELD - CENTER-OF-MASS.

If observations are made relative to a <u>true</u> horizon or gravity gradient, the longitudes and latitudes obtained would not be, in general, the spherical center-of-figure ones. The longitudes and latitudes obtained after the stochastic deflection of the vertical is taken into account are called the pseudo coordinates of the position. In LANS, corrections to convert pseudo coordinates to spherical coordinates can be generated as orthogonal polynomial expansions using pseudo-coordinates as arguments. Of course, there is no way to go from <u>observed</u> coordinates to pseudo coordinates as yet. This would require information, not presently available.

Since deflections of the vertical can occur perpendicular to the meridian as well as along it, LANS Basic Program outputs the deflections as deviations of latitude and longitude. Because the lines of longitude tend to converge, the numerical values of the longitude deflections get larger toward higher latitudes. The actual angular deviation is

$$\Delta\theta = \sqrt{(\Delta \mathrm{Lat})^2 + (\ \mathrm{Long})^2 \cos^2 \mathrm{Lat}}$$

where $\Delta\theta$ = general deflection of vertical.

The local, stochastic variations will be discussed in detail in section five.

Although a simple orbit calculation routine is included in LANS, the highest accuracy can only be obtained if the best heliocentric ephemerides for the sun, moon, earth, planets and stars are available for use. The U. S. Naval Observatory Nautical Almanac Office is the source of such data in the United States. They have available, on tape, the following required ephemerides and their published sources, if any.

a) Geocentric positions of the sun, rectangular, equatorial, 1950.0 coordinates (Astronomical Papers, Vol. XIV).

b) Geocentric equatorial, rectangular, 1950.0 coordinates of the moon, in earth radii, (U. S. Naval Observatory Circular, No. 91).

c) Heliocentric planet positions, equatorial, 1950.0, rectangular coordinates, (Astronomical Papers XII, XV (Part III), U. S. N. O. Circulars 90 and 95, Planetary Coordinates (1960-1980).

d) Stars 1950.0 mean positions (FK3, FK4, <u>Apparent Places of Stars</u> (after 1960).

e) Center-of-Mass of the Solar System (Astronomical Papers XIII IV).

The (e) entry is not available on existing tapes and for the purposes of LANS, it might be easier to generate the data rather than punching the cards to make the tape. Here are the necessary formulae. If X_o, Y_o, Z_o are the heliocentric coordinates of the center-of-mass, M_n = the planet mass in terms of the solar mass, and x_n, y_n, z_n are the heliocentric coordinates of the nth planet, then the heliocentric distance of the center-of-mass is given by

$$X_o = \Sigma \frac{m_n x_n}{(1 + \Sigma m_n)}$$

$$Y_o = \Sigma \frac{m_n x_n}{(1 + \Sigma m_n)}$$

$$Z_o = \Sigma \frac{m_n x_n}{(1 + \Sigma m_n)}$$

A table of m_n values as well as other gravitational constants may be found on p. 493 of the Explanatory Supplement. An Auxiliary Program in LANS calculates the center of mass position if x_n, y_n, z_n are known.

It was not possible during the development of LANS to fabricate any input data tapes, because master tapes were not available to the investigators and because it was not possible to predict the I/O modes that ultimately will be used with LANS. Present input for the basic program is by punch cards. Output is line printer and punch cards. Since I/O is basically a programmer's problem and not a scientific one, only the I/O necessary for testing has been considered. Tape and disk I/O are left to the choice of the users.

Finally, two further items are included. First, in outline form, the main LANS programs and their general functions are summarized. Secondly, also in outline form, the auxiliary programs are summarized. A detailed flow chart of the LANS BASIC Program is presented in Appendix 1.

Because the Empirical Correction Program and Orbit Program are adaptations of previously existing programs no detailed explanations of these will be given. This is also true of the auxiliary programs. These were developed for testing purposes. Their external functions are not included in the contract scope, but they should prove useful for future simulations.

Lunar data and constants used in LANS not referenced explicitly have been obtained from Allen (1964) or from the Explanatory Supplement (1960). Numerous other references can be found in each as well as the review by Goudas (1966).

A. MAIN PROGRAMS - LANS.

1) BASIC PROGRAM (A1)

   a. STATIONOCENTRIC COORDINATES.
   b. TOPOCENTRIC COORDINATES AND PHENOMENA.
   c. TOPOCENTRIC PSEUDO COORDINATE TABLES.

2) EMPIRICAL CORRECTION PROGRAM (A2)

   a. DETERMINES LONGITUDE/LATITUDE from observation.
   b. DETERMINES SPACEDRAFT ORIENTATION from observation.
   c. PERFORMS STATISTICAL ANALYSIS OF OBSERVATIONS.
   d. PERFORMS TIME CORRELATION ANALYSIS OF NON-RANDOM RESIDUALS.
   e. DETERMINES SHORT TERM SECULAR OR PERIODIC EMPIRICAL CORRECTIONS, IF RESIDUALS ARE NON-RANDOM.
   f. CORRECTS OBSERVED LONGITUDE AND LATITUDE ASSUMING LOCAL DEFLECTIONS NEGLIGIBLE.

3) GAUSS ORBIT CALCULATION PROGRAM (A3)

   a. FINDS SINGLE SET OF ORBITAL ELEMENTS USING THREE POSITIONS OVER LONGEST TIME INTERVAL.
   b. FINDS MULTIPLE SETS OF ELEMENTS FROM OBSERVATIONS TAKEN THREE AT A TIME.
   c. MAY BE MODIFIED TO OUTPUT EQUATORIAL ELEMENTS A_x, A_y, A_z, B_x, B_y, B_z instead of a, e, ω, i, Ω.

B. AUXILIARY PROGRAMS

1) POWER SERIES LEAST-SQUARES - Upon Option will take ORBITAL ELEMENT DATA from (A3) and supply coefficients for (A1), if time-dependent orbital elements are used.

2) ORTHOGONAL POLYNOMIAL SURFACT FIT - Takes differences between spherical longitudes/latitudes and pseudo coordinates as generated by (A1) and generates the required polynomial coefficients needed in (A3) to make corrections back to spherical coordinates.

3) LIBRATION COEFFICIENT INTERPOLATION PROGRAM - Input of appropriate β and f values gives a punched deck of libration coefficients.

4) CENTER-OF-MASS GENERATOR - Takes planetary coordinates of all planets and finds center-of-mass of Solar System.

5) CALENDAR DATE - JULIAN DATE CONVERTER.

6) NEWTON INTERPOLATION SCHEME.

Sec. 4 Empirical Test Programs and the System Operation

As mentioned previously several test progrmas were developed for use with LANS - BASIC Program. It is beyond the scope of report to explain these in detail. The methods used are standard. The following references are recommended.

1) Empirical Test Program (A2)

   (a) Statistical Analysis - Hafley, Wilkinson, Fardo (1964)

   (b) Time Series Analysis - Korn and Korn (1961)

2) Orbit Program - Dubyago (1961) (A3)

3) Other programs which are modifications of standard Burroughs Library routines are documented within the program listings. (see sec. 7).

(A) Empirical Data Program

Of course, the empirical test program actually requires real data for meaningful results. Here is a description of its operation.

1) Observed altitudes/azimuths are inputed along with stationocentric object coordinates.

2) Upon option either the longitude/latitude or appropriate coordinate constants are found by least-squares. The least-squares solution uses "normal" trigonometric transformation equations.

The longitude/latitude option is called MOON. Using option space/craft, coordinate constants can be found either in the case of a rotating spacecraft or a stabilized spacecraft. For a rotating spacecraft two modes can be used.

a) Time-dependent observations of a single object options SINGLER and HOLDER (see section 7) must be set true (1).

b) Observations of a number of objects. Option HOLDER MUST BE FALSE. Option SINGLER SHOULD BE SET TRUE. (1), if residuals are to be analyzed. For a non-rotating spacecraft, Option HOLDER MUST BE SET FALSE. Input of observations of a single object will not process correctly. Option SINGLER SHOULD BE SET TRUE, if residuals are to be analyzed.

Note, however, although ALPH, DELT are used as label names, the coordinates input for the spacecraft option may either be earth equatorial coordinates (α, δ) or ecliptic (λ, β). The constants ℧, ε, etc. will then refer to that plane. In either case, the equatorial spacecraft constants that are obtained will enable a transfer from altitude/azimuth to the earth equator (α, δ) or ecliptic coordinates (β, λ) to be made.

3) The least-square coefficients are then used to determine altitude/azimuth residuals.

4) The residuals are then tested for correlations with three quantities

   a) time

   b) altitude

   c) azimuth.

5) A cross-correlation check is then performed.

6) If the data is non-random then the data is fit by least-squares to either a

   a) power-series polynomial or a

   b) fourier series

7) Upon option, correction formulae (orthogonal polynomials) with arguments of spherical longitude/latiutde can be used to correct derived coordinates to spherical longitude/latitude. The coefficients for these polynomials are generated from the orthogonal surface fit program using data given by the basic program.

(B) Orbital Element Program

1) Observations of Planetary objects are input.

2) The Gauss method is used to solve for orbital elements or sets of orbital elements.

3) If a single set of elements is desired then these can be used directly in the LANS Basic Program.

4) If several sets of elements are obtained, these are used as data into the power series least-squares program. The output of the least-squares is then input to LANS Basic Program.

(C) A Suggested System Operation

In the previous sections of this report, we have discussed the most general aspects of LANS. We have pointed out that LANS can be modified to fit situations other than that of lunar surface navigation. In section 3, the lunar problem was outlined, but no specific mode of operation was suggested. But a number of data exchange and interchanges between programs and areas of study are necessary before LANS can be used to the fullest advantage. At the end of section 3, the LANS programs were listed. Using the notation of that listing, a general chart of data flow is shown in figure 12. It can be seen that LANS has many interactions with observational research as well as theory. Thus, although the necessary computer programs have been developed specifically in this contract, much work on the necessary parameters and constants is needed before celestial navigation can be done accurately from the lunar surface (see Sec. 6 for estimates of <u>present</u> uncertainties). Assuming the necessary data input has been obtained, <u>one</u> possible use for the system will be described.

Suppose a series of photographs have been taken from the lunar surface including the horizon and some celestial objects. The local gravitational vertical is known and all of the angular measurements including that of the horizon have been referred to this vertical. The zero azimuth direction is arbitrary.

The approximate coordinates of the camera position are input to LANS-Basic Program along with the brighter planet and navigation star (heliocentric) data over the required interval.

The selenocentric orbital elements of an orbiting instrument module are also put in LANS Basic. The specific dates and times (U.T.) of the photographic exposures are put in the Julian date converter. The corresponding dates and the Universal Time - Ephemeris Time correction are input. The appropriate options are set and approximate - altitudes/azimuths and calendar dates are obtained. These are compared to observations and identifications made.

Another option in LANS Basic is set and, for increments of longitude and latitude around the approximate coordinates, a data deck giving the corrections to the center-of-figure, spherical coordinates is obtained. This deck is then put into the orthogonal fitting routine. The resulting deck from the orthogonal fit is then put into the empirical data program.

Another option is set and selenocentric coordinates (center-of-mass) at the appropriate times are obtained and input into the empirical data program along with the altitude/azimuths (referenced to the gravitational zenith) obtained from observations.

The empirical data program is then set and the spherical apparent (pseudo coordinates) longitude/latitude are obtained by least-squares as well as the apparent azimuth of the north point. The altitude/azimuth residuals are then analyzed by autocorrelation techniques. (Suppose a periodic trend is found). The empirical Fourier coefficients obtained are then used to correct the observations. The corrected observations are then put back in and a new longitude and latitude are obtained. The residuals are tested again. If they are random, another option is set and the apparent camera position is corrected back to a spherical, center-of-figure system.

If desired, the entire process can be repeated using the horizon as a reference. But suppose some of the exposures show instrument module images. Using standard formulae, the altitudes and azimuths in the gravity system can be transformed to a center-of-mass system in right ascension and declination. (Pseudo-coordinates should be used for longitude and latitude. Any coordinate (systematic) effects, including deflections of the vertical, should be removed).

The right ascensions and declinations and other appropriate coordinates are entered into the orbit calculation program. A set of orbital elements are obtained. If these are significantly different from previous ones then local variations of the vertical may be contributing.

Without going into detail, the satellite images or a series of exposures of circumpolar stars may be used to derive deflections of the vertical. The path of the moon's north celestial pole relative to earth 1950.0 equatorial coordinates can be obtained by option from LANS Basic. The local "land" slope can be obtained as well.

Other examples could be given. But until the instrumentation and methods to be used are worked out more completely, these serve no useful purpose in this report. In anticipation of a number of uses for LANS, a number of additional options were included, but not mentioned above. The program listings should be consulted for further details.

Sec. 5 ESTIMATES OF LOCAL DEFLECTIONS OF VERTICAL ON THE MOON

One source of error in lunar surface navigation is the <u>stochastic</u> local deflection of the vertical. We have made no attempt to derive rigorous theoretical formulae for this, since in practice it must be determined empirically. However, we may put limits on the expected contribution using some simplified models.

Moulton (1914, p. 118) has stated that the deflection of the vertical λ due to a hemispheric "bump" is

$$\tan \lambda = \frac{(1/2\pi\sigma)r}{\pi\sigma 2R - \sigma 1 r} \approx 1/2 \frac{\sigma 1 r_o^3}{\sigma 2 R} \approx \lambda \qquad \text{.....(1)}$$

where R is the radius of the moon, r is the height of the "bump", σ_1 is the mean density of the "bump", σ_2 is the mean density of the sphere. For a "hole" $-\sigma_1 = \sigma_2$.

It can be shown that the deflection of a buried spherical mass is

$$\tan \lambda = \frac{r_o^3(\sigma 2 - \sigma 1)d}{(x^2 + d^2)^{3/2}} \Big/ \frac{r_o^3(\sigma_2 - \sigma_1)x}{x^2 + d^2 + \sigma_1 R} \qquad \text{.....(2)}$$

where d is the depth of the mass r_o the radius of the radius of the mass, x is the distance <u>along</u> the surface from the observer of the mass, σ_1 is the mean lunar density and σ_2 is the mean density of the buried mass.

For a mass near the surface $d = r_o$ and for an observer near the edge of the mass with $r_o < R$, $x = r_o$

$$\tan \lambda = 2.8 \frac{(\sigma_2 - \sigma_1)}{\sigma_1} \qquad \frac{r_o}{R} \times 3 \frac{\sigma 2}{\sigma 1} \quad \frac{r_o}{R} \qquad \text{.....(3)}$$

For a mountain approximated by a 45° cone, the mass is reduced by 1/2 so

$$\tan \lambda \sim 1/4 \frac{\sigma_1 h}{\sigma_2 R} \qquad .....(4)$$

For $\sigma_1 = \sigma_2$

$$\tan \lambda \sim 0.25 \ (h/R) \qquad .....(5)$$

The <u>limits</u> to a conical mountain of any slope α can be found by finding the deflections for an inscribed hemisphere and a circumscribed hemisphere. For a mountain of height h, slope α, and an observer x distance from the mountain base, it can be shown that

$$\frac{1}{2} \frac{\sigma_1}{\sigma_2} \frac{h^3 \sin^3\alpha}{R(h\tan\alpha + x)} < \tan\lambda < \frac{1}{2} \frac{\sigma_1}{\sigma_2} \frac{h^3 \tan^3\alpha}{(h\tan\alpha + x)R} \qquad .....(6)$$

or at x = 0

$$\frac{1}{2} \frac{\sigma_1}{\sigma_2} \frac{h}{R} \frac{\sin\alpha}{\cos^2\alpha} < \tan\lambda < \frac{1}{2} \frac{\sigma_1}{\sigma_2} \frac{h}{R} \qquad .....(7)$$

For small slopes or for craters approximated by shallow inverted cones,

$$\frac{1}{2} \frac{\sigma_1}{\sigma_2} \frac{h}{R} \sin\alpha < |\lambda| < \frac{1}{2} \frac{\sigma_1}{\sigma_2} \frac{h}{R} \qquad .....(8)$$

Consider the following crater model. A central peak X_o from the outer base with slope α_3 the rampart of the crater walls are a distance X_1 from the central peak. The inner crater walls have a slope of α_2 and the outer walls slope α_1. X_2 is distance from the crater wall-central peak intersection to the central peak. Then

$$\tan\lambda_{min} \leq \tan\lambda \leq \tan\lambda_{max}$$

$$\tan\lambda_{max} = \frac{1}{2} \frac{\sigma_1}{\sigma_2} \frac{X_o \tan^2\alpha}{R\cos\alpha} \qquad .....(9)$$

$$\tan\lambda_{min} \approx \frac{1}{2} \frac{\sigma_1}{\sigma_2} \frac{1}{R} \left[\frac{X_o \tan^2\alpha}{\cos\alpha} - \frac{2X_1^3 \tan\alpha_2 \sin\alpha}{X_o^2} \right.$$

$$+ 2 \frac{x_2^3 \tan^3\alpha_3 \sin^3\alpha_3}{x_0^2} \Big]$$

For small and equal slopes

$$\tan \lambda_{min} \approx \frac{1}{2} \frac{\sigma_1}{\sigma_2} \frac{1}{R} \left[x_0 \alpha^2 - \frac{2 x_1^3}{x_0^2} \alpha^2 + \frac{2 x_2}{x_0^2} \alpha^6 \right] \quad \text{....(10)}$$

For "average" craters $\alpha \sim 0.2$, $x_2 = 0.1\ x_o$, $x_1 = 0.9\ x_o$

and $$\lambda_0 \approx \frac{1}{2} \frac{\sigma_1}{\sigma_2} \frac{x_0}{R} (0.04) \quad \text{..... (11)}$$

We do not claim that these equations can be used to calculate <u>actual</u> deflections, but they can be used to evaluate limits of accuracy. Four cases have been considered and are enumerated in the following tables.

a) For small conical craters or hills (outer slope ≈ 0.2) at edge

Height or depth	max $\|\lambda\|$	min $\|\lambda\|$
500 ft.	10"	2"
1000 ft.	20"	4"
2000 ft.	40"	8"
4000 ft.	80"	16"
1 mi.	102"	20"
2 mi.	204"	40"

b) For large craters with central peaks and outer slope 0.2

radius	max $\|\lambda\|$	min $\|\lambda\|$
1 mi.	100"	4"
2 mi.	200"	8"
3 mi.	300"	12"
4 mi.	400"	16"
5 mi.	500"	20"
10 mi.	1000"	40"
30 mi.	3000"	120"
50 mi.	5000"	200"
100 mi.	10000"	400"

c) For buried spherical iron mass just touching surface and observations a radius away from the surface tangent

radius	$\|\lambda\|$
10 ft.	2"
50 ft.	9"
100 ft.	18"
500 ft.	90"
1000 ft.	180"
1 mi.	900"

d) For fairly steep mountain ∿ 45°

radius	max $\|\lambda\|$	min $\|\lambda\|$
500 ft.	10"	7"
1000 ft.	20"	14"
1 mi.	102"	70"
2 mi.	200"	140"
3 mi.	300"	210"
4 mi.	400"	280"

Sec. 6 ACCURACY OF EPHEMERIS DATA FOR THE LUNAR SURFACE AS GENERATED USING LANS

In the previous section we have attempted to find by several approximations, the lower limit to the stochastic component of the deflection of the vertical. The situations considered are oversimplified and experimental results must be obtained before an estimate of how large an uncertainty is due to random variations of the vertical. On the basis of the numbers derived in Section 5, estimates of this error will be mentioned later in this section. First, however, some estimates of other contributing uncertainties must be made.

Because of the large number of transformations involved in LANS it has been virtually impossible to do rigorous error analyses of major parts of the system. Many constants employed are not "universal" and are poorly known. Hence, a real test for this system is to perform <u>accurate</u> observations from the lunar surface and compare them with the computations. However, since no rigorous <u>error</u> analysis can be performed, the test program results obtained by variations of sensitive quantities must be used. It should be stressed that the shortcomings are numerical only. No approximations were made that would make LANS <u>system</u> or process limited, at least, for operations that involve non-relativistic mechanics. We have concluded that for the <u>Basic Program</u>, LANS is <u>data-limited</u>. On the basis of the data available at the completion of this final report and expected future refinements, we have attempted to make estimates of various sources of error. The contributions listed are uncertainties of topocentric altitudes/azimuths referenced to a particular zenith and center.

a) Positions at center-of-mass of moon.

Best Heliocentric Data	± 0".005
Average Orbital Elements	± 0".10
Preliminary Orbit	±10".0
"Best" Operational Estimate	± 0".05

b) Center-or-Mass to Center-of-Figure (Goudas, 1966)

Maximum:

Estimated Error (current)	± 5'
Estimated Error (Post-orbiter)	± 30"
Estimated Error (Post-Apollo)	± 0".1
"Best" Operational Estimate	± 40"

Note: The correction here is not a parallax error, but is a systematic deflection of the vertical.

c) Libration Constants (Eckhardt, 1965)

Maximum Error Due to β, f Uncertainty	± 30"
Estimated Error (Post-orbiter)	± 1".0
Estimated Error (Post-Apollo)	± 0".1
"Best" Operational Estimate	± 10"

d) Surface Shape ("geoid")(Goudas, 1966, Bray and Goudas, 1966).

Maximum (Triaxial ellipsoid)	± 50"
Maximum (8th order Expansion)	± 30' (!)
Maximum (only 4th order coefficients)	
a. Polar Regions	± 2'
b. Equatorial Regions	± 30"
"Best" Operational Estimate	± 30"

Note: The estimates in (d) do not include the difference between geographic spherical coordinates and center-of-mass spherical coordinates as this is already included in (b).

e) Gravity Field (Relative to center-of-mass)(Goudas, 1966 - Homogeneous body).

Maximum (Triaxial Ellipsoid)	± 20"
Maximum (8th order)	± 10' (!)
Maximum (4th order)	± 30"
a. Polar Regions	± 30"
b. Equatorial Regions	± 10"
"Best" Operational Estimate	± 50"
"Best" Operational (Post-orbiter)	± 5"

f) Local Stochastic Vertical Deflections

Possible Uncertainty

a. Mountainous Areas	± 100"
b. b. Plains	± 20"
c. Plains (surface undulations)	± 100"
"Best" Operational Estimate	± 50"

g) Spherical Topocentric System (geoid or surface corrections are ignored)

Gravity Reference (Estimate)	± 1'	Systematic
Surface Reference (Estimate)	± 4'	

The total r.m.s. uncertainty for center-of-figure spherical coordinate references can be compared with center-of-mass spherical coordinate references.

Estimated Operational Errors. (Projected)

Source of Error	Gravity and Center-of-Mass	Surface (perfect horizon) and Center-of-Figure	
(a)	±0".05	(a)	±0".05
(b)	---	(b)	± 40"
(c)	± 10"	(c)	± 10"
(d)	---	(d)	± 90"
(e)	± 50"	(e)	---
(f)	± 50"	(f)	± 100"
		(g)	∿ ± 130"
Total r.m.s.∿	± 60"		

r.m.s. ± 1' arc r.m.s. ± 2' arc

If the general deflections of the vertical are ignored, the errors are systematic as well as stochastic.

$\Delta\phi$	$\pm 1'$ arc		$\Delta\phi$	$\pm 4'$ arc
r.m.s.	$\pm 1'$ arc		r.m.s.	$\pm 2'$ arc

Finally, even if the geoid or gravity field are considered, current probable errors on the known constants for expansions to the fourth order

r.m.s.	$\pm$ 2'		r.m.s.	$\pm$ 8'

These represent the uncertainties of reference to a spherical system at the respective coordinate origins.

Because of the smaller lunar radius, one nautical mile on the moon has a spherical arc of 4' not 1' as on the earth. For navigational purposes, the current set of constants are probably suitable, but not so for geodesy. But outlook is not as gloomy as it might seem, especially in view of research now in progress or being planned. As expected a gravitational reference system is superior to a surface system. This is fortunate for celestial mechanical purposes, but is a disadvantage for geological or geodetic uses. By the time, LANS is to be used extensively, it is probable that the situation will be much better.

A hybrid system which will probably be the most useful in practice employs a gravitational zenith as a reference line, but has its origin at the center-of-figure. Such a system would not use a horizon reference and thus eliminate the errors of the geoid. This system would suffer, however, if the position of the center-of-figure cannot be determined accurately.

Estimated Errors
Gravitation Zenith, Center-of-Figure
(Local Deflections of Vertical are ignored)

Maximum (Current)	Post-Orbiter	Post-Apollo
±5' of arc	±30" of arc (?)	±10" of arc

Again it must be stressed that the figures given are estimates. It is important that the actual errors be evaluated as soon as possible, by lunar surface observations.

For system tests, we have chosen constants which we believe to be the best available, but the ALGOL programming flexibility permits a change when other constants become available. As an example of the differences which can result for divergent choices of the β, γ parameters for the moments of inertia, the following table of positions of Mars as seen in selenocentric coordinates is given below.

MARS POSITIONS

γ = 0.00010	β = 0.00063 γ = 0.00022 (near tabular singularity)	γ = 0.00032
ΔR.A. = 221".6 ΔDec = +32".2 ΔS.T. = -15.8 sec	ΔR.A. = +41".8 ΔDec = - 5".8 ΔS.T. = + 0.6 sec	ΔR.A. = -267".5 ΔDec = - 33".1 ΔS.T. = +16.5 sec
β = 0.00060	γ = 0.00020 β = 0.00066	β = 0.00063
ΔR.A. = +20.5 sec ΔDec = -304.5 sec ΔS.T. = +0.9 sec	ΔR.A. = -22.6 sec ΔR.A. = +312.2 sec ΔR.A. = -0.9 sec	ΔR.A. = 0.0 sec ΔR.A. = 0.0 sec ΔS.T. = 0.0 sec

β IS THE EQUATORIAL MOMENT PARAMETER
γ IS THE POLAR MOMENT PARAMETER

Sec. 7 Programming Details

LANS has been programmed in Extended ALGOL 60. The basic LANS program has been diagramed in Appendix 1. However, it is not necessary to know the details of these programs in order to use them, provided details are available of the appropriate data formats. In this section, these details are presented for the three main programs. Input instructions for the auxiliary programs are also given.

The files for the empirical data reduction program have all been declared internally, but for the basic program and the orbit program, no files were declared internally in original versions, but calls and formats for card reader (CR), card punch (CPH), and line printer (LP) have been left in. These formats, especially for the basic program, are not elaborate in anticipation of changes to magnetic tape or disk for I/O operations. Headings have been omitted for a number of options of output, but the data obtained are fully described in this section so no confusion should result.

A. Basic LANS Program

The following is a description of data input required for operation of the basic LANS program in its present form. It be noted that some of the Boolean options are mutually exclusive. If any one of these is set true all of the others must be set false. These are listed below. If not already declared, files CR, CPH, and LP must be declared.

SUNNY, POLAR, EARTHY, NAGAV.
CHECK, EPHEM, LONLAT.

Some options can be used only if other options are set true, these are:

PUNCHOPT - LONILAT
LONILAT - LONLAT

ORBCOMP, MOONEL, PLANEL, DIRECTO — ORBFL or STATION
MEANCAT - STAR
RISESPHERE - LONLAT
HELIO - EARTHY
BARBY - READINOPT

All other options either are ignored, if not needed, or can be used independently. All Boolean options are L6 formats, right adjusted. The data cards required are:

1) L6 - Option HOMOG, one card, if true when GENERAL is true will compute gravity field coefficients from geoid coefficients.

2) L6 - Option TRANITE, one card, if true when GENERAL is true will generate gravity field using center-of-mass longitude, latitude, and radius to compute gravity parameters rather than using center-of-figure coordinates.

3) L6 - Option SUNNY, if true, solar coordinates (an appropriate field of zeros) must be input later as a planet. Then, if SUNNY, zero longitude sidereal time and zero longitude solar time are listed as a function of date. (One card).

4) L6 - Option DATEFIX, if true, calendar conversion of Julian date is written. Otherwise numerical Julian date is printed. (one card).

5) L6 - Option INVERT, one card, if true, the heliocentric coordinates of the station must be used instead of the stationocentric coordinates of the SUN.

6) L6 - Option PUNCHOPT, one card, may be true only if LONILAT IS TRUE. If true, punches cards for use in orthogonal polynomial fit program. If false, the actual pseudo-coordinates are printed out in the order - sphere - gravity-surface.

7) L6 - Option READINOPT, one card, if true, allows a large set of constants to be read in directly later in the data deck. (see comment (21) for list of constants.)

8) L6 - Option UTETCON, one card, if true, it must be followed by a card giving the Ephemeris Time minus Universal Time correction in seconds free-field format). This option will interpolate tabulated quantities for difference between U.T. and E.T. If false, E.T. is used. If true, U.T. is used.

9) L6 - Option POLAR, one card, if true, the 1950.0 equatorial (earth) coordinates of the lunar north pole are obtained. Geoid - gravity constants are omitted, as are planet-sun-moon positions if this option is used. The libration constants, however, must be read in as usual followed by a card in free-field format giving the starting Julian date and the number of days for which pole positions are desired after that date (see items after (20)).

10) L6 - Option EARTHY, one card, if true the program runs as usual except geocentric coordinates are obtained. Positions are corrected for everything but nutation. READINOPT should be set; otherwise, the constants used for computation of the zenith will be those for the moon. General should not be set true unless the geoid-gravity constants for the earth are input.

11) L6 - Option BARBY, one card, if true, precession constants may be read into the program directly.

12) L6 - Option FOLLOWUP, one card, if true, then a new set of options and data follows and the program will repeat until FOLLOWUP is false.

13) L6 - Option NAGAV, one card, if true, the local sidereal time, and object-hour angle, will be printed out along with the LONGITUDE (ANGULAR AND TIME UNITS) and LATITUDE. LONLAT must be true. (see (15)).

must also be true; if true and PUNCHOPT false the spherical and pseudo-longitude and latitudes (gravity and geoid) are printed out; if true and PUNCHOPT true a set of <u>four</u> data decks are punched. The order of decks are:

a) LATITUDE geoid - LATITUDE sphere = f_1 (LAT sphere, LONG sphere)

b) LONGITUDE geoid - LONGITUDE sphere = f_2 (LAT sphere, LONG sphere)

c) LATITUDE gravity - LATITUDE sphere = f_3 (LAT sphere, LONG sphere)

d) LONGITUDE gravity - LONGITUDE sphere = f_4 (LAT sphere, LONG sphere).

Each of these decks are to be run through the orthogonal fit program to obtain four coefficient sets which in turn are used by the empirical data program. In addition to the data decks, the following information is printed out.

a) spherical longitude and latitude

b) total deflection of the vertical

c) normalized values of local center-of-figure distance and gravity magnitude. If the number of latitudes, n and number of longitudes, m for which the four data decks are desired are such that $n \times m > 1023$, then PUNCHOPT is automatically set false and the pseudo-coordinates are printed out.

15) 2F10.1, 2I5, 7L6, one card giving the following:

a) Julian Date of First data point of first data set (FIRST).

b) Julian Date of last data point of first data set (LAST).

c) Number of Tabulations <u>per day</u> for the desired output (PERDAY).

Note: (FIRST - LAST) x PERDAY = All must be less than 80 to prevent array overflow.

d) Number of objects, both planetary and stellar for which data is to be generated. (ENDRUN).

e) Boolean DECIS - if true geocentric lunar cooridnates given in earth radii will be converted at astronomical units.

f) Boolean SPHERE - if true, topocentric corrections will be computed for spherically symmetric geoid and gravity fields. Output order is: sphere (center-of-figure) - sphere (center-of-mass) - sphere (center-of-figure).

g) Boolean ELLIPSE - if true, topocentric corrections will be computed assuming triaxial geoid and gravity field. Output order is: sphere (center-of-figure) - gravity (center-of-mass) - geoid (center-of-figure).

h) Boolean GENERAL - fi true, topocentric corrections will be computed using spherical harmonic expansions of the geoid and gravity field. OUTPUT order is sphere-gravity-geoid.

i) Boolean CHECK - if true, ecliptic and nodal coordinates are read out.

j) Boolean EPHEM - if true, a standard selenocentric ephemeris will be generated including coordiantes, distances (for planets) and radial velocities as a function of time.

k) Boolean LONLAT - if true, (with LONILAT FALSE) a topocentric ephemeris in altitude/azimuth referenced to a spherical, gravity, and geoid respectively, is printed.

16) L6 - Option HELIO, one card, if true, one set of primary coordinate data can be omitted from input (see 23).

17) "Free-field", two integers (TERMA, TERMB) which indicate, respectively, the order of the geoid and gravity expansions for which coefficients will be available and read in if GENERAL is true.

18) "Free-field" - Two integers (TERMC, TERMD) which indicate number of terms to be truncated from the series expansion in the calculation of the geoid and gravity field, in case GENERAL is true.

19) "Free-field" - six Real Numbers - "for topocentric corrections"

a) LONNY - smallest longitude value for tables

b) LATTY - smallest latitude value for tables

c) BIG - longitude interval for tables > 0

d) AIG - latitude interval for tables > 0

e) BIGG - largest longitude value for tables

f) AIGG - largest latitude value for tables.

If [(LONNY - BIGG)/BIG x (LATTY-AIGG)/AIG]> 1024 PUNCHOPT is set false and "LONGITUDE - LATITUDE GRID TOO LARGE FOR ARRAY" is printed. If only one longitude - latitude position is to be used then set LONNY = AIGG = longitude and LATTY = BIGG = latitude and AIG = BIG = 1.0. (one card)

20) L6 - Option INTRER and integer (NNI), one card, if true and NNT > 0 then NNI cards must follow giving specific Julian dates for which data in the range between FIRST and LAST is desired. The format for these cards, if included, is F12.6.

The items (1) to (20) must be present in every data deck, even if they are not used for a particular option. The next items are listed in proper order, but may or may not be used for a particular option. The next items are listed in proper order, but may or may not be present according to the options set in (1) - (20).

21) If READINOPT then the following cards must come after (20).

a) Free-field, one card, (WOMEGA) giving the rotation rate in radians per day.

b) Free-field, one card, (CENX, CENY, CENZ) giving the x, y, and z displacements of the center-of-figure relative to the center-of-mass.

c) Free-field, one card, planet (earth) radii to astronomical units conversion factor. (ERTOAU).

d) Free-field, one card, satellite (moon) radii to astronomical units conversion factor. (LR to AU).

e) Free-field, one card, mean radius (RZERO) and height above mean radius (HZERO) for topocentric corrections.

f) Free-field, one card, sine (SIE) and cosine (COE) of the obliquity of the ecliptic for epoch other than 1950.0 may be input here if desired, These are used in precession and libration calculations.

g) Free-field, one card, seven numbers (A0, B0, C0, CAPA, CAPB, CAPC, M) giving the X, Y, Z, triaxial semi-major axes, the X, Y, Z moments of inertia, and the mass respectively. These are used for the ELLIPSE option.

22) If GENERAL then the following cards must come after either (20) or (21).

a) Cards with TERMA coefficients in 6E10.2 format giving the spherical <u>geoid</u> expansion.

b) If <u>not</u> HOMOG, then cards with TERMB coefficients follow in 6E10.2 format. If TRANITE these coefficients must be for an expansion in center-of-mass longitudes/ latitudes, instead of center-of-figure longitudes/latitudes.

If LONILAT, (22) is the last data to be included for input.

23) Primary Data Sets - Three groups, unless HELIO is true, in which case set (c) below is omitted. Each group has a set of cards, the first of which has, free-field format, the order of interpolation (N), the number of days time argument of the data (DAYS), the Julian date of the first data point (FIRS) and the Julian date of the last data point (LAS). Be sure that (LAS - FIRS)/ DATE <80. The remaining cards have, in <u>free-field format</u>, X, Y, Z coordinates, respectively, for each day between (LAS-FIRS) at intervals of DAYS data. The three sets should have X, Y, Z coordinates read in the following order (INPUT [I, II, J])

a) Planetocentric (geocentric) coordinates of the Sun (II=0)

b) Heliocentric coordinates of center-of-mass of solar system (II=1)

c) Planetocentric (geocentric) coordinates of moon (II=2).

23) L6 - Option STATION, one card, if true, it must be followed by the following set of cards. If it is false then (25) follows immediately after (24).

If STATION THEN

a) 4L6, one card, four Boolean Options

1) ORBCOMP - if true, orbital elements will be used to generate station positions.

2) MOONEL - if true, orbital elements referenced to the lunar center-of-mass may be used. It is assumed that these elements use a Kepler's constant such that the orbital semi-major axes are in lunar radii units.

3) PLANEL - if true, orbital elements referenced to a planet center-of-mass may be used, provided the elements give distances in planet radii not astronomical units.

4) DIRECTO - if true, X, Y, Z heliocentric coordinates can be input directly <u>without</u> disturbing the primary data set.

b) L6 - Option READELEM, one card, if true only a single set of orbital elements will be used. If false, a set of coefficients giving the time-dependent orbital elements must be put in.

c) If ORBCOMP then a set of cards follow (b):

A) If <u>not</u> READELEM then coefficients are input.

one card - F12.4 - Julian date of time-epoch (E[0])
one card - F12.8 - obliquity of ecliptic in degrees. (ECL).
six cards - 3R12.8 - on each card the constant term, first and second power coefficients of <u>one</u> orbital element. (ACOFF, BCOFF, CCOFF).

The order is a, e, i, ω, Ω, T.

B) If READELEM then INSTEAD of (A) use:

one card - 6F12.8 - giving orbital elements in order a, e, i, ω, Ω, T in degrees. (E[I]).
one card - F12.8 - ecliptic obliquity in degrees. (ECL).

It should be noted that the appropriate cards for (A) or (B) can be obtained directly from the orbit calculation program (B) or from the orbit calculation program as processed by the power-series least-squares program (A).

d) If MOONEL OR PLANEL, one card, F12.8, giving the appropriate Kepler constant.

e) If DIRECTO, items (c) and (d) are omitted. Instead, a set of (LAST-FIRST) x PERDAY (= ALL) cards must be put in. On each card in 3R12.9, the X, Y, Z 1950.0 coordinates are entered.

25) In 6E12.3 format, a series of cards. The first field gives the mean axial inclination (INC), the remaining fields give the libration - nutation coefficients. A deck in the appropriate format and order may be obtained dd by using the coefficient β, λ interpolation program. The order of the first forty-five coefficients is as follows.

(COEFF[J,I])

a)	10 τ coefficients	(J = 1),
	2 (F - D)	(I = 1)
	(L')	
	(L - L' - D)	
	(L - 2D)	
	(L)	
	2 (L - L' - D)	
	(2(L - D) - L')	
	2 (L - F)	
	2 (L - D)	(I = 10)
	5 blank fields	(I = 11 to I = 15)

b)	15 - Iσ coefficients	(J = 2)
	2 (F - D)	(I = 1)
	2F	
	(L - 2F)	
	(L - 20)	
	L	
	2L	(I = 6)
	9 blank field	(I = 7 to I = 15)
c)	15 ρ coefficients	(J = 3)
	2 (F - D)	I = 1
	2F	
	(L - 2F)	
	(L - 2D)	
	L	
	(L - F)	I = 6
	9 blank fields	(I = 7 to I = 15)

If option POLAR is true then items (22), (23), and (24) must be omitted. Then (25) is followed by a single card, which in free-field format, the Julian date for which the computations of the pole position are to start and the number of days for which positions are desired thereafter. If POLAR is true, this card is the last item in the deck and any of the following items must be omitted.

26) Secondary Data Sets - card sets, <u>ENDRUN</u> in number, with following characteristics.

a) 2L6, Option's STAR, ORBEL one card.

b) If STAR is false, one card. L6, F12.2, Option RISESPHERE, integer giving the unit distance semi-diameter, in seconds of arc. RISE-SPHERE AND LONLAT are true, then rise, set, and transit times are computed.

c) If STAR is false, ORBEL is false, the cards that follow refer to objects within the solar system, but give X, Y, Z coordinates directly. If ORBEL is true, it serves the same function as STATION in (24). The cards that follow, if ORBEL is true, are identical in format and label to those used in (24, a-e) and will not be repeated here.

If STAR is false and ORBEL is false, cards (a) and (b) are followed by:

d) one card, 2A6, 2I5, 2F15.3, the first two fields contain the object name, the third field is an integer giving the order of interpolation (N), the number of days interval at which the X, Y, A data is available, the last two fields contain the Julian dates of the first and last data points which follow. (FIRS, LAS).

e) X, Y, Z date, in number and format exactly as for set (a) in (23) with free-field format but with only single array INPUT [I, O, J] being used. There are <u>no</u> sets corresponding to (b) and (c) in (23) in this part of the deck.

If STAR is ture, ORBEL is ignored, items (b), (c), (d), (e) are replaced by (b') and (c') given below.

(b') 2A6, 2I3, F7.3, 2I3, F6.2, 3F7.3, one card, the first two fields contain the star name, the next three fields contain the star <u>1950.0</u> right ascension in hours, minutes, and seconds, the next three fields contain the 1950.0 declination in degrees, minutes and seconds, the next field the stellar parallax in seconds of arc, the next field contains the stellar proper motion at 1950.0 in seconds of time of right ascension, the last field contains the declination component of the stellar proper motion in seconds of arc. (STARND[0], HRS, MINT, SECT, DEG, MINA, SECA, PIE, MUA, MUD).

(c') 2L6 - Options RISESPHERE and MEANCAT. If RISESPHERE and LONLAT are true, the time of rise, set, and transit of the star are computed. If MEANCAT is true, the 1950.0 quantities entered in (b') can be mean catalogue places, not "true" mean places. (see <u>Explanatory Supplement</u> Chapter 4 on mean and apparent places of stars.)

If FOLLOWUP is true an entirely new deck must follow. The next deck must be prepared according to the items (23) through (26) above, except (25) is omitted in the followup decks. Each "followup" begins with a single card, similar to (20), on which the Julian Day of the first data point and the Julian Date of the last data point in each followup deck is put in 2F10.1 format. Then in 2I6, the number of desired tabulations per day and the number of objects are printed. The tiems (23), (24), (26) are entered as for the first deck.

If FOLLOWUP is false, the data deck is complete.

It has not been possible in the development period to test all of the options under all data conditions. All options have been tested and barring typographical punch errors, or incorrect data handling they should operate normally. For a number of constructs, it may be desirable to reformulate some operations. Several options are necessarily slow. These should be avoided, if not absolutely essential. The worst are DATEFIX, UTETCON, and RISESPHERE. Option UTETCON is particularly time consuming since double interpolations are involved.

At the present time, the astronomical input data (X, Y, Z coordinates) is accurate to 10^{-8} astronomical units. The normal functions SIN(X), COS(X), SQRT (X), etc., are also accurate (for single precision) to about 10^{-8}. If angular arguments to an accuracy of $\pm 0\overset{''}{.}001$ arc or smaller become possible or are desirable, double precision function procedures are recommended.

B. LANS EMPIRICAL DATA PROGRAM (LUNAR)

There are basically two parts to the LUNAR program. One part calculates longitude and latitude from the moon's surface, and the other part calculates the equatorial epoch, ascending node Ω, and inclination E for navigation from a space craft.

The input data depends completely on which part of the program will be used. An initial input card containing a free field one or zero determines the part to be used.

Part One - Navigation on the Lunar Surface

1. Initial option card = 1, followed by either a zero or one in free-field format. If zero then randomness of residuals is not tested. The second figure must be one, if observations of single object are input. If the observer is spin-stabilized, the figure must be zero and coordinates of M different objects used. This sets option SINGLER.

2. Second option card determines whether data for a sphere, a geoid, or for a gravity field will be used in the orthogonal calculations for longitude and latitude. The card must contain, in free-field format, three integers which are either zero or one, where one indicates this option is to be used . The first number is for a sphere, second for a geoid, and third for gravity. If space craft (see later) is true, the first number should be one and the others zero.

3. Actual observational data - in chronological order

This data must contain all observed information for the calculation. Each observation must be on two cards, and the set of data must be followed by a sentineal card containing 80 asterisks.

Data for each observation must have the following form, where everything is free-field format:

Card 1 - Azimuth degrees, azimuth minutes, azimuth seconds, altitude degrees, altitude minutes, altitude seconds.

Card 2 - Time in Julian days, alpha hours, alpha minutes, alpha seconds, delta degrees, delta minutes, delta seconds, zero lunar sidereal time in hours, minutes, seconds. If spacecraft then alpha/delta can be ecliptic coordinates (1950.0) (λ,β) or equatorial coordinates 1950.0 (α,δ). If not spacecraft, alpha/delta must be equatorial coordinates 1950.0 (α,δ). There must be at least three observations and there cannot be more than 500 observations.

4. Sentinal card described above.

5. Coefficient Deck for orthogonal polynomial calculation of latitude and longitude. This deck must be in the format punched by Burroughs program PTS-062. There will be four parts to the deck - from 4 distinct runs of PTS-062. They must be arranged in the following order: Geoid - latitude coefficients, Geoid - longitude coefficients, Gravity - latitude coefficients, Gravity - longitude coefficients.

6. Tolerance to be used in random test. This must be a number in free-field format, and should be less than three sigma estimated instrument error for the original observations, but larger than the estimated variance of the initial least-squares reduction.

7. Order of polynomial fit in random test.

This must be a positive integer in free-field format. The number must be less than ten.

8. Order of Fourier fit in random test.

This must be a positive integer in free field format. The number must be less than ten.

<u>Part Two</u> - Spacecraft navigation option.

1. Initial option card = 0, followed by either a zero or one in free-field format. If zero then randomness of residuals is not tested. See note on first data card described in Part 1. This sets option SINGLER.

2. Second option card determines whether the spacecraft is rotating or not, and whether one object is used as the observation point to determine alpha and delta. This card must contain two numbers in free-field format. The numbers must be either zero or one, where one indicates that this option is the one to be used. The first number indicates if rotation is assumed. (OPTION ROTATE) The second indicates whether one or more objects are used for alpha, delta observations. (OPTION HOLDER)

3. Actual observational data - Part 1 - in chronological order.

This data must contain azimuth and altitude observations. The data must be in free-field format, with one observation per card. A sentinal card must follow the data and must contain 80 asterisks. The data on each card is assumed to be:

Azimuth in degrees, azimuth minutes, azimuth seconds, altitude degrees, altitude minutes, altitude seconds, time in Julian days.

There must be at least three observations, but no more than 500.

4. Sentinal card described above.

5. Either an omega value to be read in or a set of values to be used for a randomness test depending on the rotate option read in previously.

If ROTATE is true, a tolerance value, polynomial order value, and a Fourier order value on three separate cards must be read in. See the description for input data 5, 6, and 7 under the Lunar Surface option.

If ROTATE is false, a card containing a number which will be Ω in the calculations must be input. The number must be in free-field format.

6. Either a single card containing alpha, delta information, or a complete set of alpha, delta information depending on the option set on the Second option card.

If only one observation is made for alpha and delta, the card must contain in free-field format, the following information:

Alpha in hours, alpha minutes, alpha seconds, delta degrees, delta minutes, delta seconds.

If several observations are made, a card must be input for each observation with a sentinal card as described above, following the set of observations. This type of data card must contain, in free-field format, the following information in chronological order:

Time in Julian days, alpha hours, alpha minutes, alpha seconds, delta degrees, delta minutes, delta seconds. At least three, but no more than 500 values should be input.

7. Tolerance value for random test; see part 5 in Lunar section.
8. Polynomial order value for random test; see part 6 in Lunar section.
9. Fourier order value for random test; see part 7 in Lunar section.

The LANS empirical data program has as its most important procedure a modified statistical program. It is called LEASTSQ. The description is a summary of the uses and needs of LEASTSQ, a procedure which performs least-squares and other analyses of statistical data. The procedure LEASTSQ is a modification of the Westinghouse Regression program. (Research Report 64-1C4-362-R1).

The purpose of the modifications was to make the program a procedure and yet allow communication with the procedure.

Communication is accomplished by declaring certain variables globally, changing the card reader file to a desk file, and saving "answers" in three globally declared arrays.

The procedure is specialized such that no extra "header" cards are needed, therefore eliminating the transformation and polynomial options available in the program, and also without the plotting facilities.

The following variables must be declared globally, and set appropriately before entering LEASTSQ:

INTERGER OPT 1, OPT 2, OPT 3, OPT4, OPT 5, OPT 6, OPT 7, OPT 8, OPT 9, OPT 10, OPT 11, OPT, 12, OPT 13, REPSS, NOLAG, M, N, Q, H; ARRAY ANS1, ANS2 [0:30], ANS3 [0:500]; FILE READER DISK SERIAL [20:500] (2, 30);

These declarations have the following meanings and implications:

M = number of observations

N = number of independent variables

Q = N + number of dependent variables

H = total number of input items per observation

READER is a file containing the observations. These are to be read unedited ("*" format) on N cards. On each card, in order, are the following:

1) dependent variable

2) independent variable

3) " "

4) " "

The OPT 1 - OPR 13, plus REPSS and NOLAG are options controlling the flow through the program. The most basic linear regression assumes all of these = 0; to "set" an option set it equal to 1.

Some options cannot be used because of the modifications.

Some that <u>can</u> be used are:

OPT 8 for printout of correlation matrix
OPT 11 for printout of correlation coefficient information

Others may be possible.

The ANS1, ANS2 and ANS3 arrays contain the only "answers" after the LEASTSQ analysis, excepting whaever printed information has been written.

ANS1 contains the regression coefficients such that

> ANS1 [1] has the coefficient for the first independent variable, ANS1 [2] for the second, and so on.

ANS2 contains the corresponding standard deviations for each regression coefficient.

ANS3 contains the Y residuals for each observation.

The current limits, which can be controlled by a change in the global declarations are:

1. No more than 30 independent variables are allowed change by altering upper bound of ANS1 and ANS2.
2. H cannot exceed 30. Change this by increasing the record length in the READER file declaration, as well as ANS1 and ANS2 consequently.
3. No more than 500 observations possible. Change by increasing the upper bound of ANS3.

The input is assumed to be arranged in READER such that m observations are read in, one record at a time.

Note: If any changes are made in the LUNAR program, which uses LEASTSQ, care must be taken to preserve the global variables m, n, q and h as being for the use of the LEASTSQ procedure.

C. GAUSS ORBIT PROGRAM

The program uses the Gauss method as outlined by Dubyago (1961). A number of options are available. Orbital elements may be determined using the extreme observations (in time) or using all the available observations taken three at a time. Only observations from one station can be used. Cards required are the following:

File declarations required are CR = card reader, CPH = card punch, LP = line printer.

1) One card, free-field, number of observations (N>3).

2) One card, L6, option SQUARE, if true orbital elements (time-independent) are punched on cards in the correct format for input to LANS. If false, card output must be used as input for the polynomial least-square.

3) One card, L6, option RESIDCAL, if true the right ascension/declination residuals are calculated and printed out, followed by the observation number,

4) One card, L6, option XERO, if true, the rectangular elements A_x, A_y, A_z, B_x, B_y, B_z are put out instead of the normal a, e, ω, i, Ω elements.

5) One card, L6, option GROUNDER, if true, the topocentric coordinates of the ground station are used instead of solar positions. This makes it possible to solve for satellite orbital elements, provided the appropriate Kepler constant has been input.

6) One card, L6, option PLANEL, if true, a different Kepler constant can be used and the positions of one satellite may be used to determine the orbit of another. PLANEL should be true, if GROUNDER is true.

7) One card, 3L6, options INVERT, SPACET AND LONGEST,

a) INVERT - if true, X. Y, Z coordinates of the observed object must be input, instead of the normal X, Y, Z solar coordinates. The orbital elements obtained are those of the station.

b) SPACET - if true spacecraft coordinate analogs of R.A. (α) and Dec. (δ) can be put in.

c) LONGEST - (must be true if SQUARE is true) if orbital elements found are those using the two extreme (in time) observations and one near the mean interval.

8) If SPACET is true, one card, free-field, is next with the following information:

a) rotation rate in radians/day (WWW)

b) longitude of descending node (WWV)

c) obliquity of ecliptic (inclination) (FFF)

d) Julian date epoch of (a), (b), (c). (TZERO)

9) One card, free-field, giving the Julian date (arbitrary) of a time near the mid-point of the set of observations. (TE)

10) Obliquity of ecliptic (for earth) for the epoch to which the observations refer, one card, free-field (EE)

11) If PLANEL is true, then on one card, free-field, the semi-major axis (in astronomical units) which changes the <u>solar</u> Kepler constant to that for the particular planet is input, (Kplanet = Ksun $a^{3/2}$) (AAAA).

12) If <u>not</u> SPACET, one card, free-field with the topocentric qualities as defined in the <u>American Ephemeris and Nautical Alamanac</u>, but in special units.

a) the longitude of the station expressed in fractions of a day $15° = 1^{hr}$, $24^{hrs} = 1$ Day (LAM)

b) the topocentric radius (expressed in astronomical units) times the cosine of the latitude, (DELXY)

c) the topocentric radius (in A.U.'s) times the sine of the latitude (DELTAZ)

13) A set of N cards, free-field format.

a) right ascension of Ith observation (EPH [I,1]) in hours

b) declination of Ith observation (EPH [I,2]) in degrees

c) Julian date at 0^h U.T. of the Ith observation calendar date (EPH[I,3])

14) One set of N cards, free-field format.

a) If SPACET then read in the Universal times of the N observations (in decimals of a day). (UT[I]).

b) If not SPACET then read in the zero-longitude (at 0^h U.T.) sidereal time (in hours), and the Universal times (hours) of the N observations. (SDT[I], UT[I]).

15) If not GROUNDER, a set of cards in the following order, free-field format.

a) one card, the number of days interval between the time arguments of the solar or planetary X, Y, Z data that is to be used. (QRS)

For each observation, four X, Y, Z coordinates (of the sun or observed planet) are required to be input so that interpolation to the time of observation may be sufficiently accurate. If QRS is equal to one day (see (a)), then the Julian date of the second X, Y, Z position of the every four should be equal to the Julian date of each observation. If QRS is not equal to one day, the X, Y, Z positions should be chosen so that the observation date falls between the second and third of each set.

b) a set of N card groups, free-field. The Julian date for each second X, Y, Z position goes on the first card, followed by four cards with X, Y, Z positions themselves, selected as described above.

(JULIO[I], SUNC1[I,J], SUNC2[I,J], SUNC3[I,J], SUNC4[I,J]).

This ends the data required for the orbit calculation program.

D. LIBRATION COEFFICIENT PROGRAM.

File declarations CR = card reader and CPH = card punch are needed. Data input consists of a single card, free-field format, which gives a β value (see Sec. 3) and an f value. The order is β first and f second in normal use, but the coefficients generated should be checked by making a run with the order reversed.

E. POLYNOMIAL LEAST-SQUARES.

All files are declared. The data arrangement of the original program is described in Burroughs Technical Bulletin MRS-134. This has been modified twice since 1964. The first modification has been described by Hoyle (1964). A reprint of his description is included with the program deck. The only modification we have made in addition to Hoyle's I/O are the following Boolean options which must be included in each data deck (For LANS, there are six consecutive ones):

a) one card, L6, option PUNCHOPT, if true, the output will be on cards and if P = N = 2 then the deck will be ready to be put in LANS. Coefficients up to P + 1 order are included,

b) one card, L6, option REPET, if true, a regression of y on x as well as x on y can be obtained without changing the data order.

c) one card, 5L6, options STEPN, INTEG, DIFFL, SELFC, FOLLOW. For LANS, STEPN ← TRUE, INTEG, DIFFL, SELFC, normally set false, and FOLLOW set true, for all but the last data deck,

d) one set of data cards (For LANS there will be six separate data decks),

e) group of data should end with a blank card.

f) one card, I2, X1, I2, giving the order of iteration and highest included power of X. (For LANS both of these should be set to 2).

F. CALENDAR DATE CONVERTER.

Files CR = card reader, LP = line printer need to be declared.

a) one card, free-field, integer giving the number of dates to be converted. (R)

b) R cards, free-field, (don't forget the quotation marks for the alpha variables) with following fields per card.

a) alpha variable, abbreviation of the month - JAN, FEB, MAR, APR, MAY, JUN, JUL, AUG, SEP, OCT, NOV, DEC, are allowed (MON).

b)	integer giving		day	(DY)
c)	integer	"	year	(YR)
d)	"	"	hour	(HR)
e)	"	"	minutes	(m)
f)	real	"	seconds	(s)
g)	alpha	"	type of time used, abbreviation.	

G. ORTHOGONAL POLYNOMIAL PROGRAM.

This program is run without modification. For information on operation see Burroughs Bulletin PTS-063.

H. CENTER-OF-MASS PROGRAM

Files CR = card reader and CPH = card punch must be declared. Input data required are:

a) one card, free-field, integer giving the total number of center-of-mass positions required. Note: all planetary coordinates to be used must be subtabulated to the same time epoch and interval before putting in this program. (NN<65).

b) nine card groups. First card, free-field format, of each group is a real number giving the mass fractions of one planet relative to the sun (see Explanatory Supplement). This first card is followed by NN cards giving the planet heliocentric coordinates X, Y, Z per card in 3E20.11 format.

I. TWO DIMENSIONAL NEWTON INTERPOLATION PROGRAM. For Z = f(x,y).

DATA INPUT only:

a) one card, L6, option PUNCHOPT, if true data is punched out in format 6E1Z.3

b) one card. free-field. integer giving number of data sets to be processed. (Q)

c) one card, free-field, inter giving the number of different values to be interpolated from each data set. (L)

d) one card, free-field, two integers, N and M giving the number of X values, and the number of Y values respectively.

e) one card, free-field, two integers, NN<N-1 and MM<M-1 giving the number of differences for X adn Y respectively.

f) in format 6E10.2, N values of the x variables

g) in format 6E10.2, M values of the y variables

h) in format 6E10.2, Q x N x M values of fq(x,y) function.

j) L cards, free-field, giving x, y values for which new f(x,y) are to be found.

This concludes the brief data-input descriptions. For further details, the program listings and flow charts should be consulted.

SUMMARY

In the preceding sections we have outlined the structure and philosophy of the Lunar Astronomical Navigation System (LANS). Although the present accuracy is not quite sufficient for surface geodetic studies, it is probably sufficient for all celestial navigation tasks. Upon completion of the lunar orbiter and surveyor programs, the physical constants should be very much improved. Thus even by the first Apollo flights very accurate ($\approx\pm 0\overset{''}{.}1$) ephemeridies might be generated, if necessary. However, a number of difficulties remain. Of particular interest are:

a) lunar libration constants, free libration and long-period forced librations,

b) amount of the center-of-figure and center-of-mass displacement, especially along the radial direction,

c) actual deflections of the vertical; accelerometer measurements in the three coordinate axes, not just the vertical component,

d) observational testing of LANS, both on the earth and from space, including simulation of navigation problems which were not possible in the present contract.

e) accurate geoid and gravity field parameters.

Item (a) can be studied in a number of ways both from the lunar surface and from the earth, but requires a concentrated effort.

Items (b) and (e) should be available as a result of the lunar orbiter and surveyor studies.

Item (c) may be available from some geological experiments now being planned, but it is important to measure the total gravity vector. If this is not included in the planning, it should be.

Item (d) is very improtant. Testing using the EARTHY option is to be carried out at MSFC. Tests from the lunar surface prior to Apollo missions are recommended, if possible. Otherwise, lunar surface operations during the first Apollo flights for LANS testing will be necessary.

Using the LANS basic program and gravity and geoid constants (up to fourth order) given by Goudas (1966) and Goudas and Bray (1966), we have generated a set of tables giving deflections of the vertical, the total gravity vector (normalized) and the normalized geoid radius as a function of spherical longitude and latitude (±75° longitude ±75° latitude in 10° steps). Also, we have generated a table of the gravity and geoid pseudo-coordinates as a function of spherical longitude and latitude. (Of course, these do not include local variations). The pseudo coordiante tables were generated for every degree step between ±60° longitude and ±60° latitude. Tapes of the pseudo-coordinate tables are available.

Copies of the all LANS programs and data will be maintained at Leander McCormick Observatory and will be available, if a disaster destroys other existing copies. We do not plan to obtain or use magnetic tape masters of the input data, however. These can be obtained from the U. S. Naval Observatory.

Comments and inquiries about LANS should be directed to Marshall Space Flight Center rather than to the observatory. It is hoped that some of this material will be available for future publication.

It is a pleasure to acknowledge the help of the following persons and groups for their contributions in the completion of this project.

Dr. Donald Eckhardt of the Air Force Cambridge Research Laboratories for his special set of libration constants.

The Computer Science Center at the University of Virginia for a grant which enabled the pseudo-coordinate tables to be generated for a large segment of the visible portion of the moon.

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Appendix 1 - FLOW CHARTS FOR BASIC LANS PROGRAM.

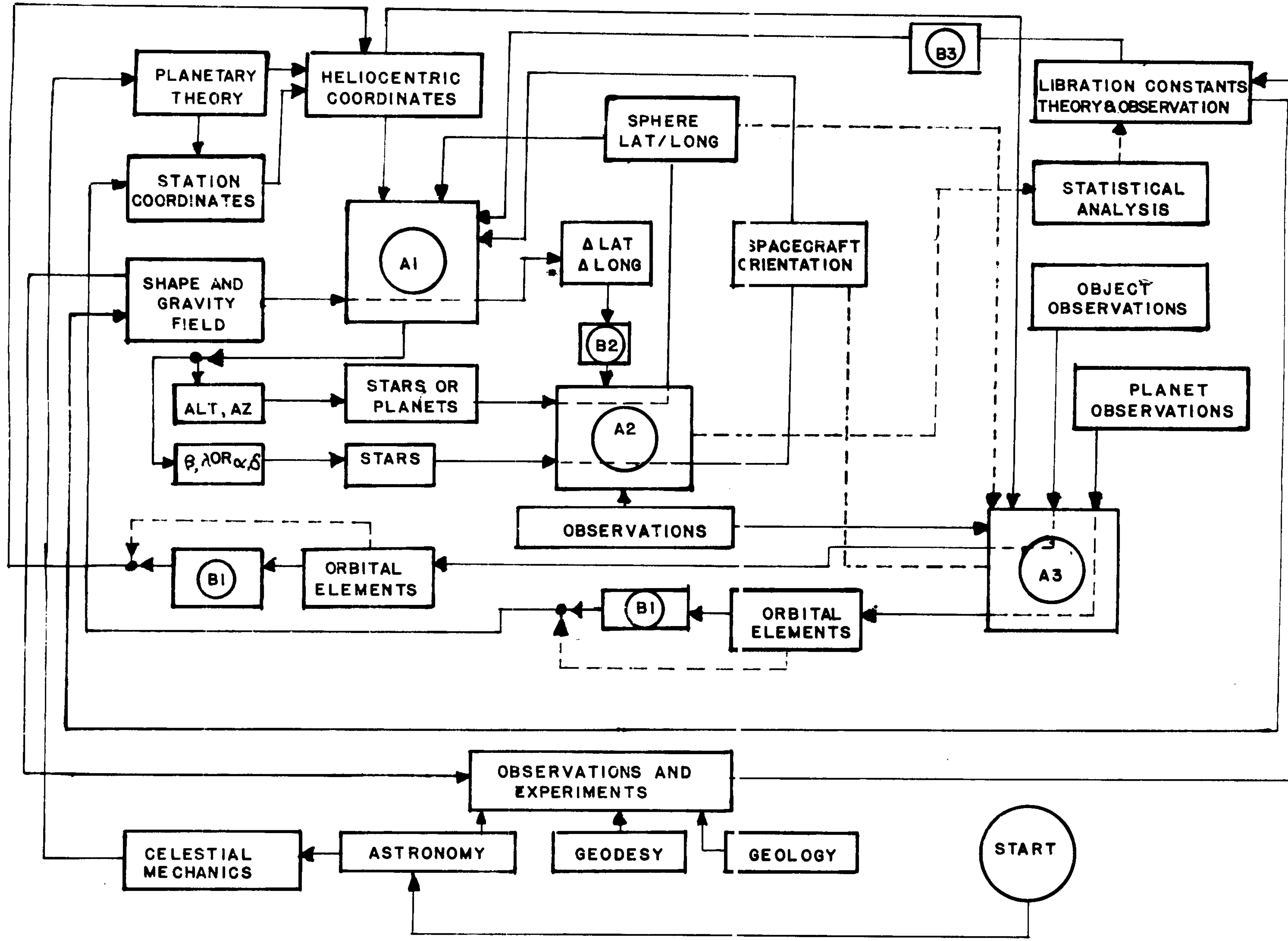
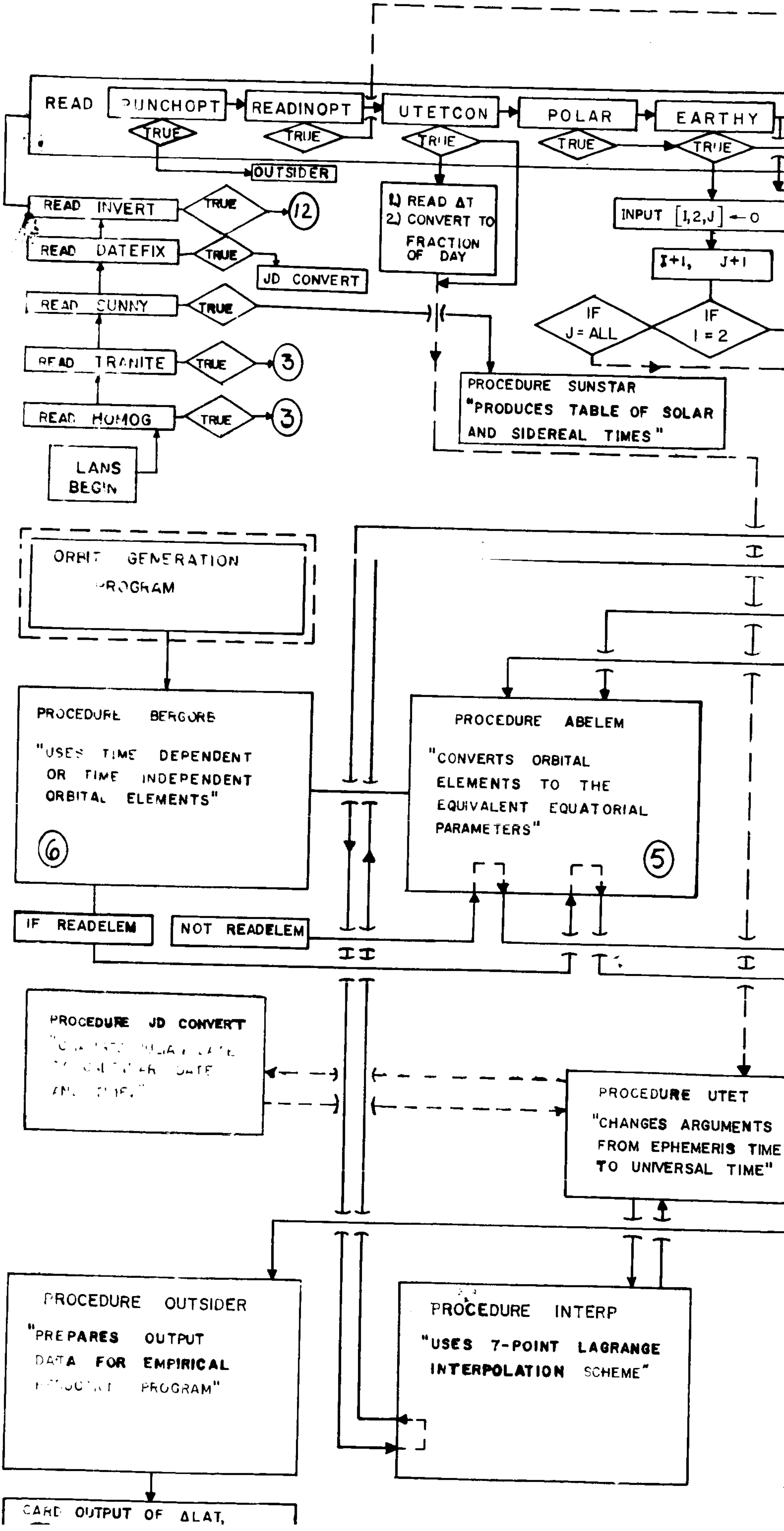
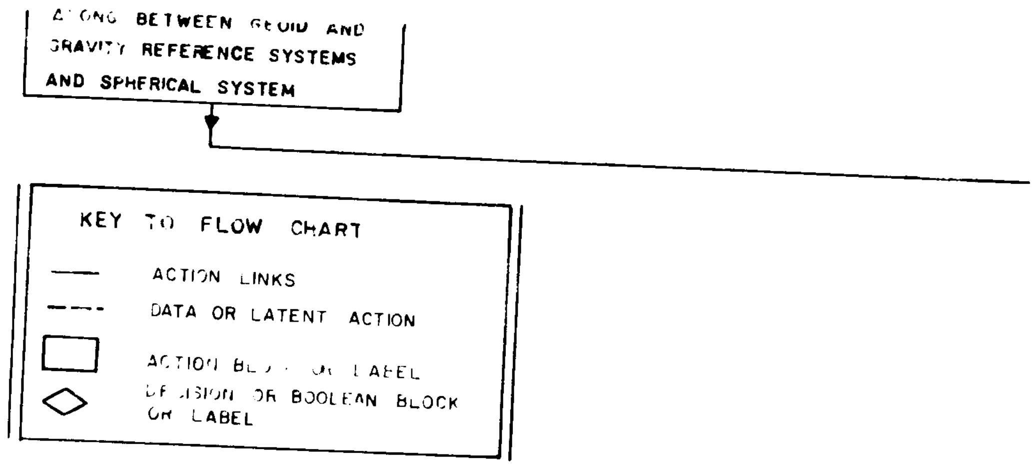


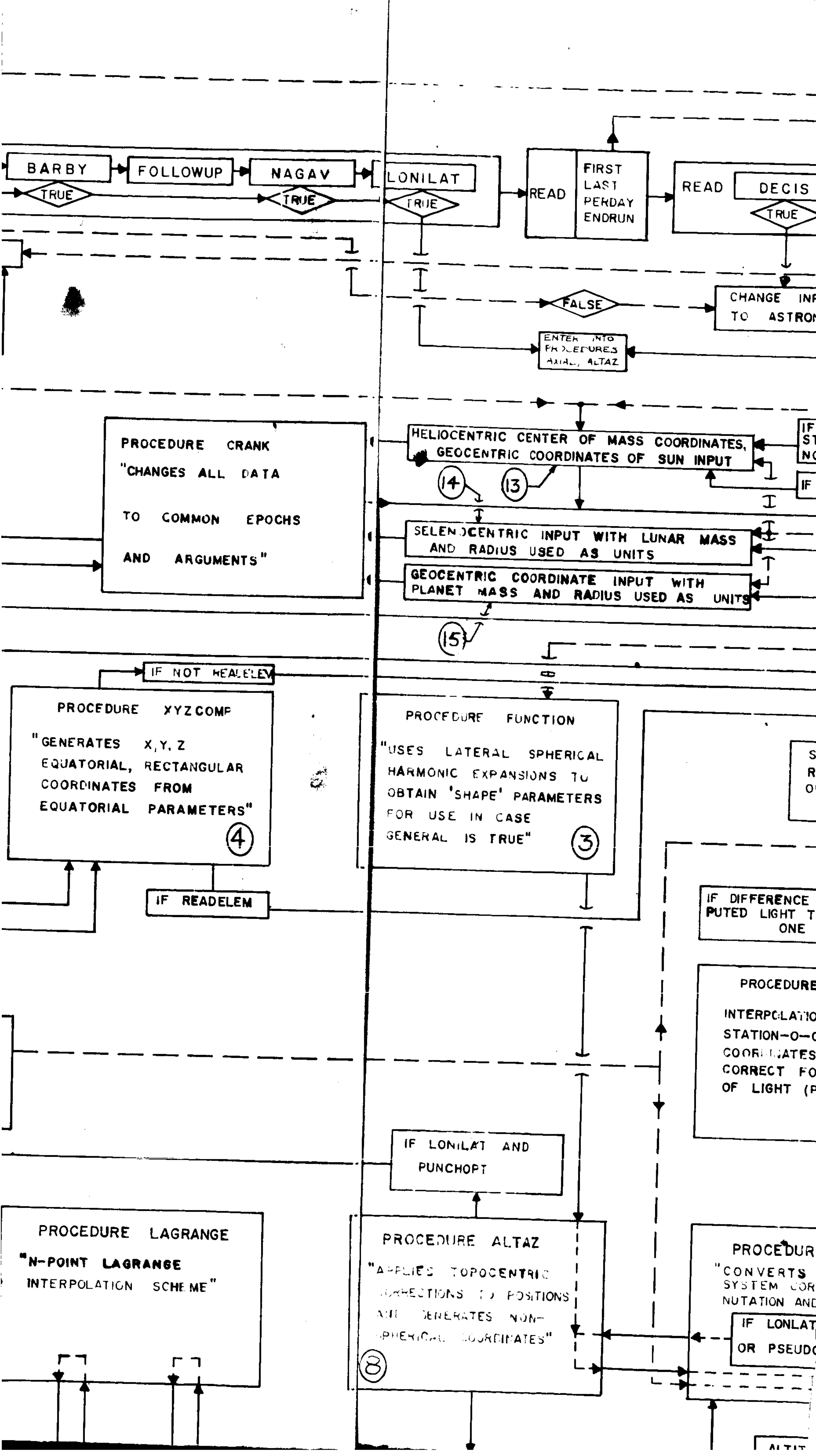
FIGURE 12 - PROGRAM-OBSERVATION-SCIENCE INTERACTIONS IN LANS



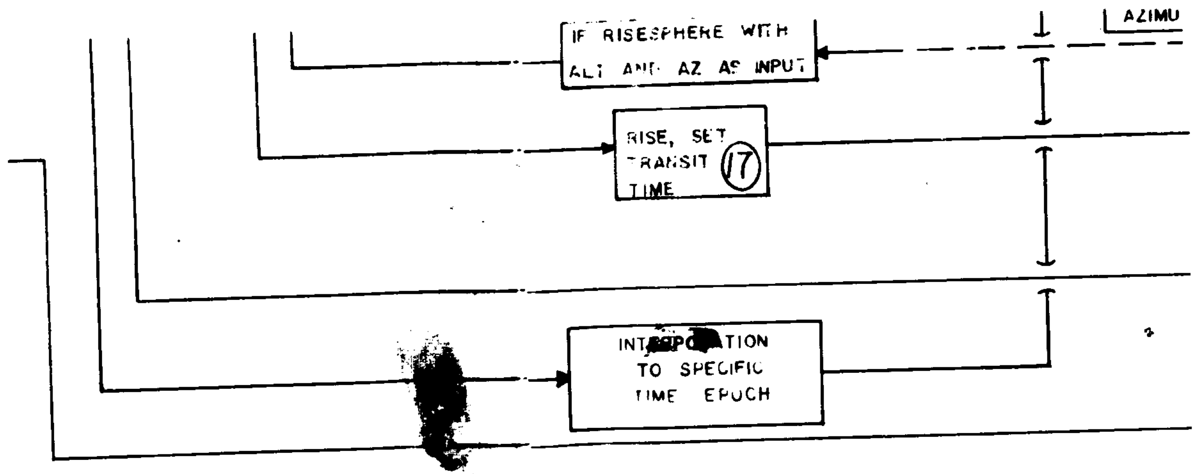
READ
PUNCHOPT
READINOPT
UTETCON
POLAR
EARTHY
TRUE
OUTSIDER
READ INVERT
12
READ DATEFIX
JD CONVERT
READ SUNNY
READ TRANITE
3
READ HOMOG
LANS BEGIN
1) READ ΔT
2) CONVERT TO FRACTION OF DAY
INPUT [1,2,J] ← 0
I+1, J+1
IF J = ALL
IF I = 2
PROCEDURE SUNSTAR
"PRODUCES TABLE OF SOLAR AND SIDEREAL TIMES"
ORBIT GENERATION PROGRAM
PROCEDURE BERGORE
"USES TIME DEPENDENT OR TIME INDEPENDENT ORBITAL ELEMENTS"
6
PROCEDURE ABELEM
"CONVERTS ORBITAL ELEMENTS TO THE EQUIVALENT EQUATORIAL PARAMETERS"
5
IF READELEM
NOT READELEM
PROCEDURE JD CONVERT
PROCEDURE UTET
"CHANGES ARGUMENTS FROM EPHEMERIS TIME TO UNIVERSAL TIME"
PROCEDURE OUTSIDER
"PREPARES OUTPUT DATA FOR EMPIRICAL PROGRAM"
PROCEDURE INTERP
"USES 7-POINT LAGRANGE INTERPOLATION SCHEME"
CARD OUTPUT OF ΔLAT,

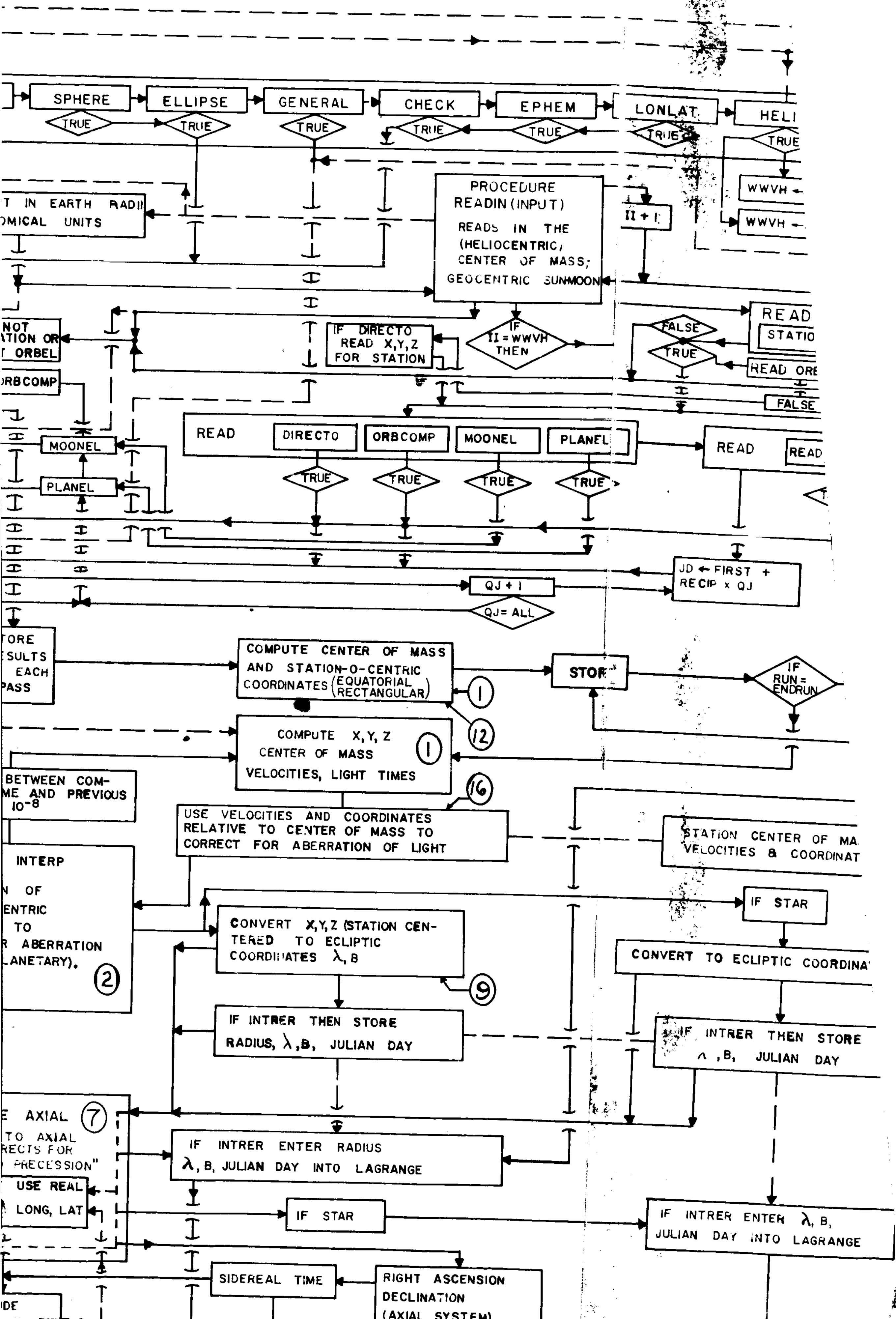


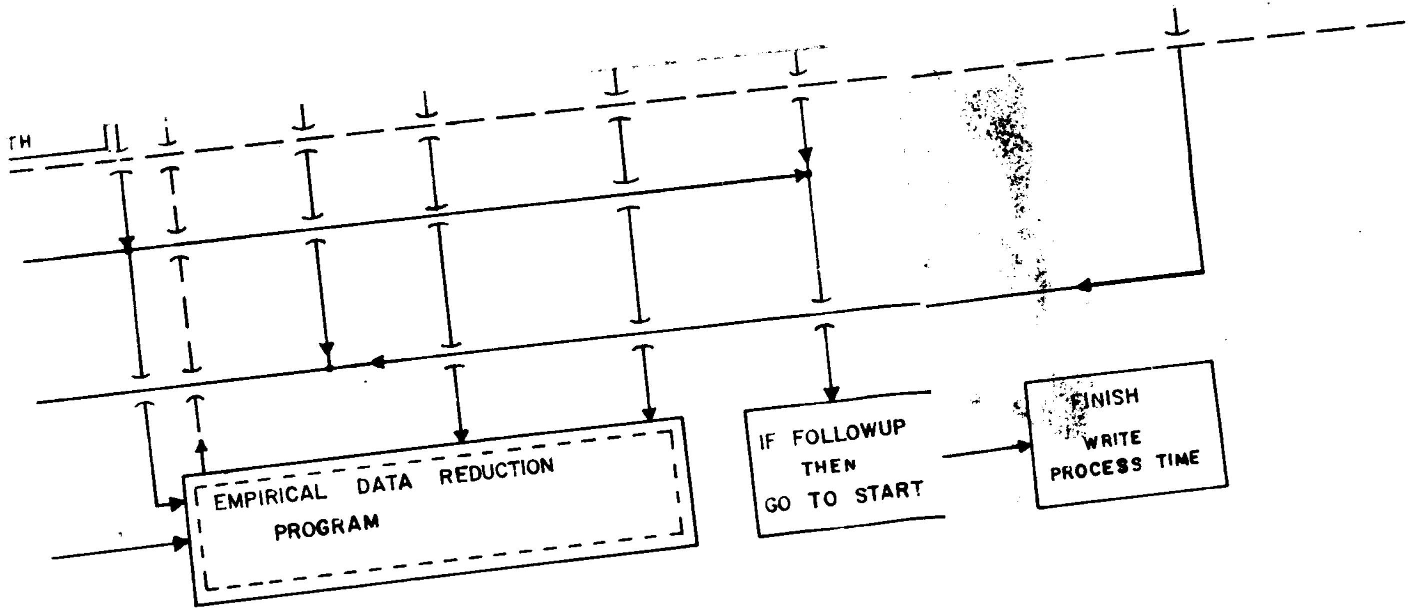
BETWEEN
AND
REFERENCE SYSTEMS
AND SPHERICAL SYSTEM
KEY FLOW CHART
ACTION LINKS
DATA OR LATENT ACTION
OR BOOLEAN BLOCK
LABEL

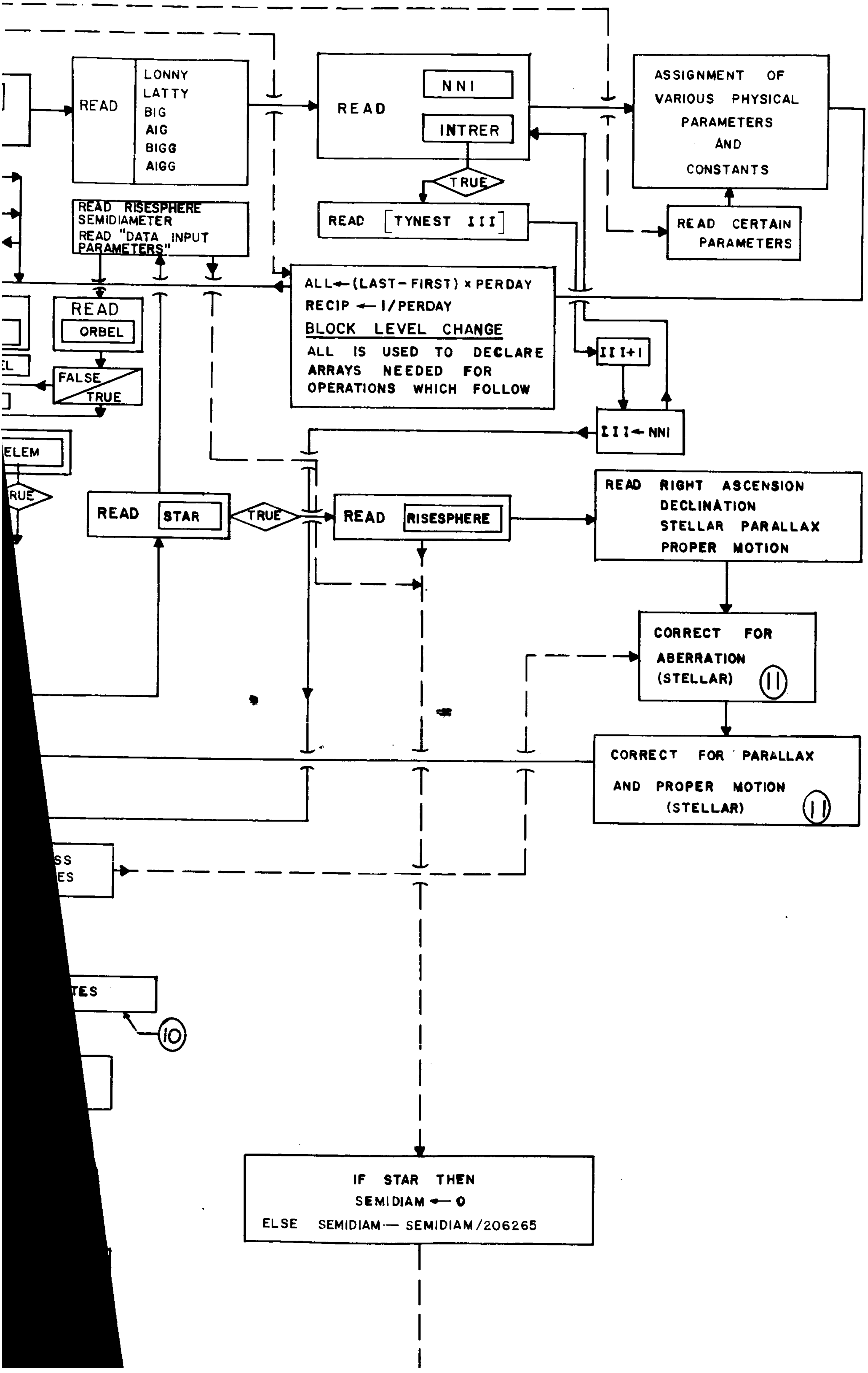


BARBY
FOLLOWUP
NAGAV
LONILAT
TRUE
TRUE
TRUE
READ
FIRST LAST PERDAY ENDRUN
READ
DECIS
TRUE
FALSE
CHANGE INF
TO ASTRON
ENTER INTO PROCEDURES AXIAL, ALTAZ
PROCEDURE CRANK
"CHANGES ALL DATA TO COMMON EPOCHS AND ARGUMENTS"
HELIOCENTRIC CENTER OF MASS COORDINATES, GEOCENTRIC COORDINATES OF SUN INPUT
14
13
SELENOCENTRIC INPUT WITH LUNAR MASS AND RADIUS USED AS UNITS
GEOCENTRIC COORDINATE INPUT WITH PLANET MASS AND RADIUS USED AS UNITS
15
IF NOT READELEM
PROCEDURE XYZCOMP
"GENERATES X, Y, Z EQUATORIAL, RECTANGULAR COORDINATES FROM EQUATORIAL PARAMETERS"
4
IF READELEM
PROCEDURE FUNCTION
"USES LATERAL SPHERICAL HARMONIC EXPANSIONS TO OBTAIN 'SHAPE' PARAMETERS FOR USE IN CASE GENERAL IS TRUE"
3
IF DIFFERENCE
PUTED LIGHT T
ONE
PROCEDURE
INTERPOLATIO
STATION-O-C
COORDINATES
CORRECT FO
OF LIGHT (P
IF LONILAT AND PUNCHOPT
PROCEDURE LAGRANGE
"N-POINT LAGRANGE INTERPOLATION SCHEME"
PROCEDURE ALTAZ
"APPLIES TOPOCENTRIC CORRECTIONS TO POSITIONS AND GENERATES NON-SPHERICAL COORDINATES"
8
PROCEDUR
"CONVERTS
SYSTEM COR
NUTATION AND
IF LONLAT
OR PSEUDO


IF RISESPHERE WITH
ALT AND AZ AS INPUT
RISE, SET
TRANSIT
TIME
17
INTERPOLATION
TO SPECIFIC
TIME EPOCH
AZIMU


SPHERE
ELLIPSE
GENERAL
CHECK
EPHEM
LONLAT
HELI
TRUE
T IN EARTH RADII
OMICAL UNITS
PROCEDURE
READIN (INPUT)
READS IN THE
(HELIOCENTRIC,
CENTER OF MASS;
GEOCENTRIC SUNMOON
WWVH ←
WWVH ←
II + 1
NOT
ATION OR
T ORBEL
IF DIRECTO
READ X,Y,Z
FOR STATION
IF
II = WWVH
THEN
READ
STATIO
FALSE
TRUE
READ ORE
FALSE
ORBCOMP
MOONEL
PLANEL
READ
DIRECTO
ORBCOMP
MOONEL
PLANEL
READ
READ
JD ← FIRST +
RECIP x QJ
QJ + 1
QJ = ALL
TORE
ESULTS
EACH
PASS
COMPUTE CENTER OF MASS
AND STATION-O-CENTRIC
COORDINATES (EQUATORIAL
RECTANGULAR)
1
12
STORE
IF
RUN =
ENDRUN
COMPUTE X,Y,Z
CENTER OF MASS
VELOCITIES, LIGHT TIMES
1
BETWEEN COM-
ME AND PREVIOUS
10⁻⁸
16
USE VELOCITIES AND COORDINATES
RELATIVE TO CENTER OF MASS TO
CORRECT FOR ABERRATION OF LIGHT
STATION CENTER OF MA
VELOCITIES & COORDINAT
INTERP
N OF
ENTRIC
TO
R ABERRATION
LANETARY).
2
IF STAR
CONVERT X,Y,Z (STATION CEN-
TERED TO ECLIPTIC
COORDINATES λ, B
9
CONVERT TO ECLIPTIC COORDINA
IF INTRER THEN STORE
RADIUS, λ, B, JULIAN DAY
IF INTRER THEN STORE
λ, B, JULIAN DAY
E AXIAL
7
TO AXIAL
RECTS FOR
PRECESSION"
USE REAL
LONG, LAT
IF INTRER ENTER RADIUS
λ, B, JULIAN DAY INTO LAGRANGE
IF STAR
IF INTRER ENTER λ, B,
JULIAN DAY INTO LAGRANGE
SIDEREAL TIME
RIGHT ASCENSION
DECLINATION
(AXIAL SYSTEM)
IDE


TH
EMPIRICAL DATA REDUCTION
PROGRAM
IF FOLLOWUP
THEN
GO TO START
FINISH
WRITE
PROCESS TIME


READ
LONNY
LATTY
BIG
AIG
BIGG
AIGG
READ
NNI
INTRER
TRUE
READ [TYNEST III]
ASSIGNMENT OF VARIOUS PHYSICAL PARAMETERS AND CONSTANTS
READ CERTAIN PARAMETERS
READ RISESPHERE SEMIDIAMETER
READ "DATA INPUT PARAMETERS"
READ
ORBEL
FALSE
TRUE
ALL ← (LAST − FIRST) × PERDAY
RECIP ← 1/PERDAY
BLOCK LEVEL CHANGE
ALL IS USED TO DECLARE ARRAYS NEEDED FOR OPERATIONS WHICH FOLLOW
III+1
III ← NNI
ELEM
RUE
READ
STAR
TRUE
READ
RISESPHERE
READ RIGHT ASCENSION DECLINATION STELLAR PARALLAX PROPER MOTION
CORRECT FOR ABERRATION (STELLAR)
11
CORRECT FOR PARALLAX AND PROPER MOTION (STELLAR)
11
SS
ES
TES
10
IF STAR THEN
SEMIDIAM ← 0
ELSE SEMIDIAM — SEMIDIAM/206265

LUNAR ASTRONOMICAL NAVIGATION SYSTEM

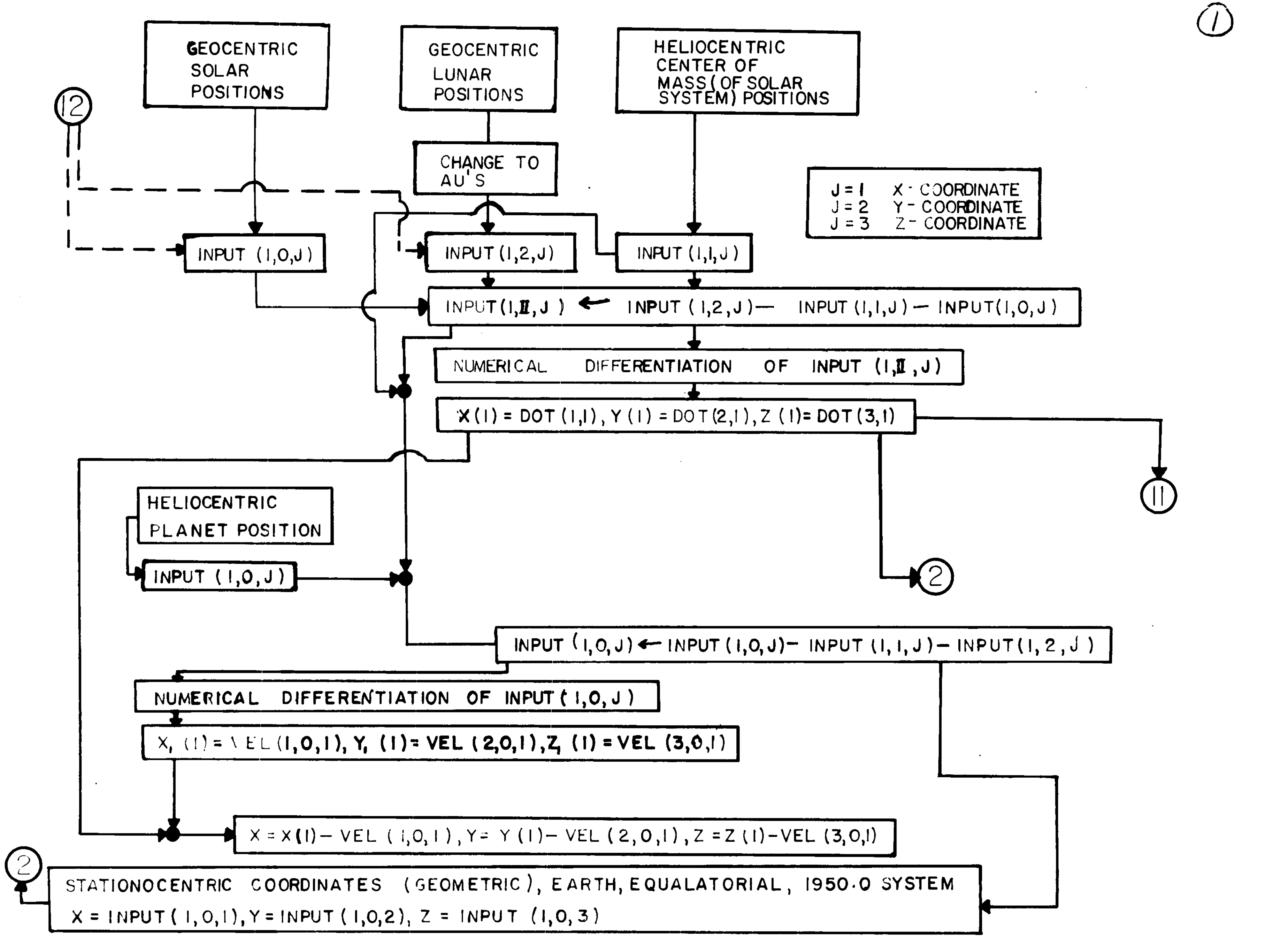
BASIC GENERATION PROGRAM

FINALIZED: 12/20/66

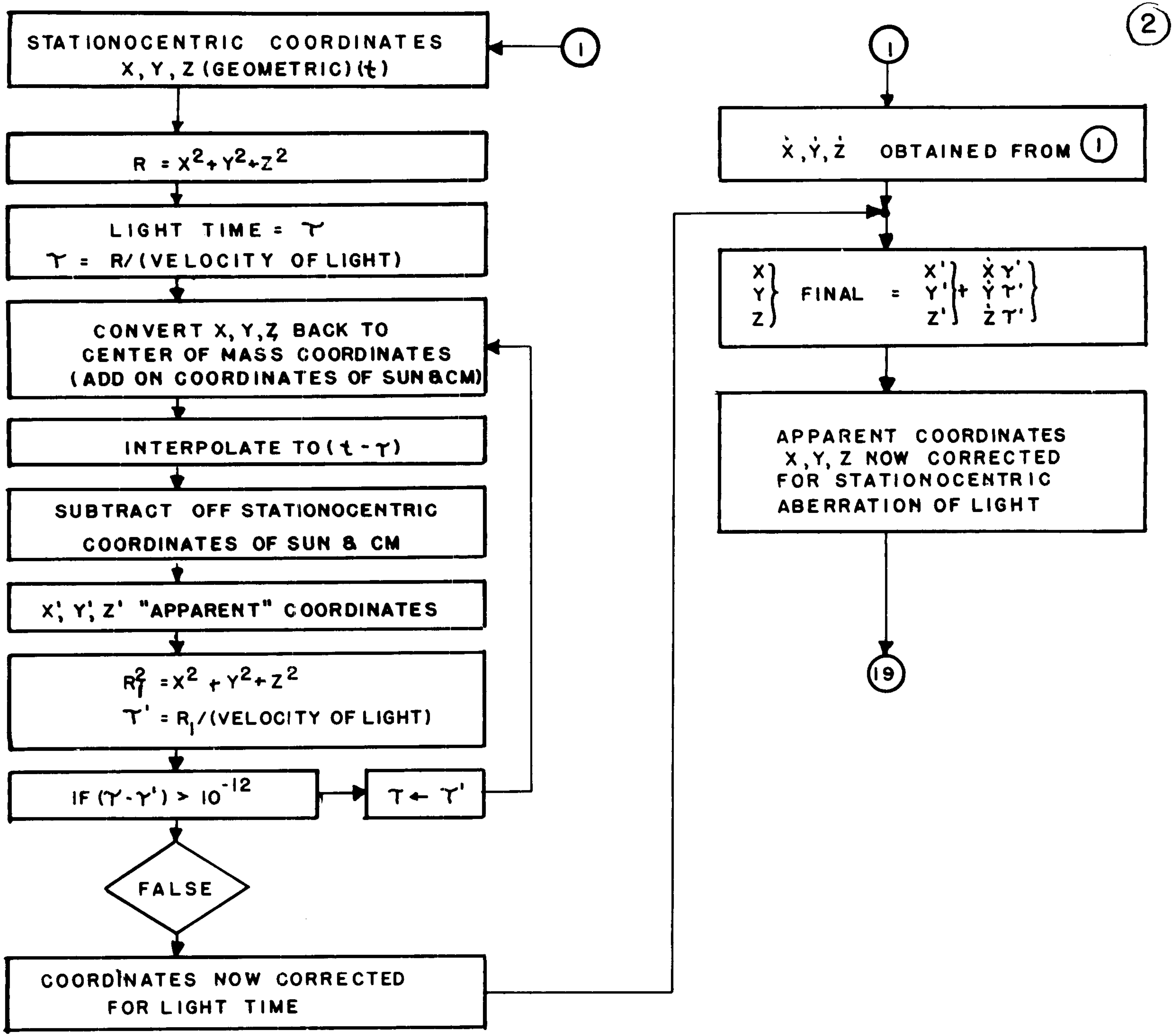
ORIGINAL DIAGRAM BY D. D. MEISEL

DRAWN BY R. A. BERG, W. D. SIEGEL, C. ROBINSON

(NUMBERS INDICATE POINTS WHERE DETAILED EQUATIONS OR PROCESSES ARE GIVEN SEPARATELY)

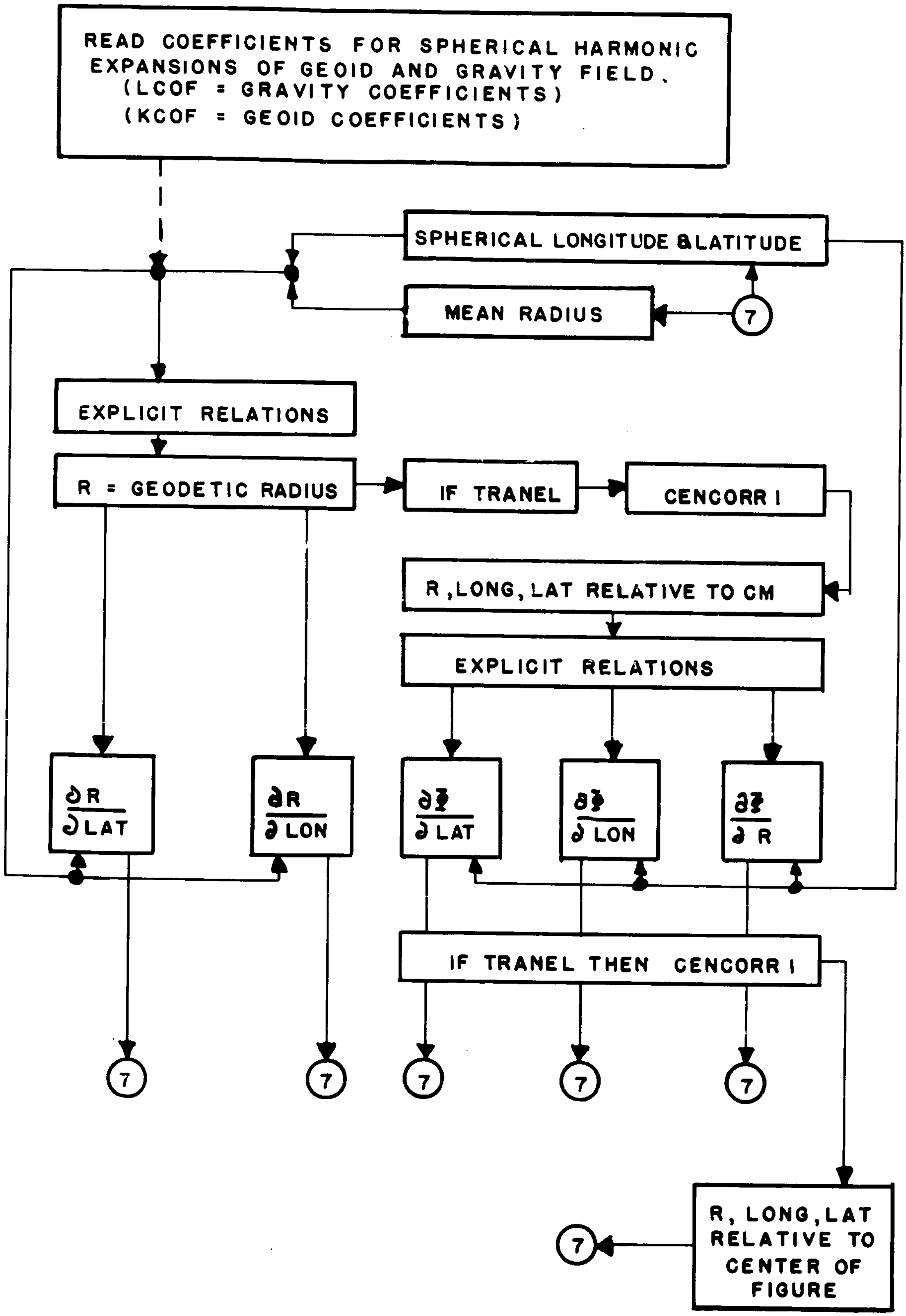


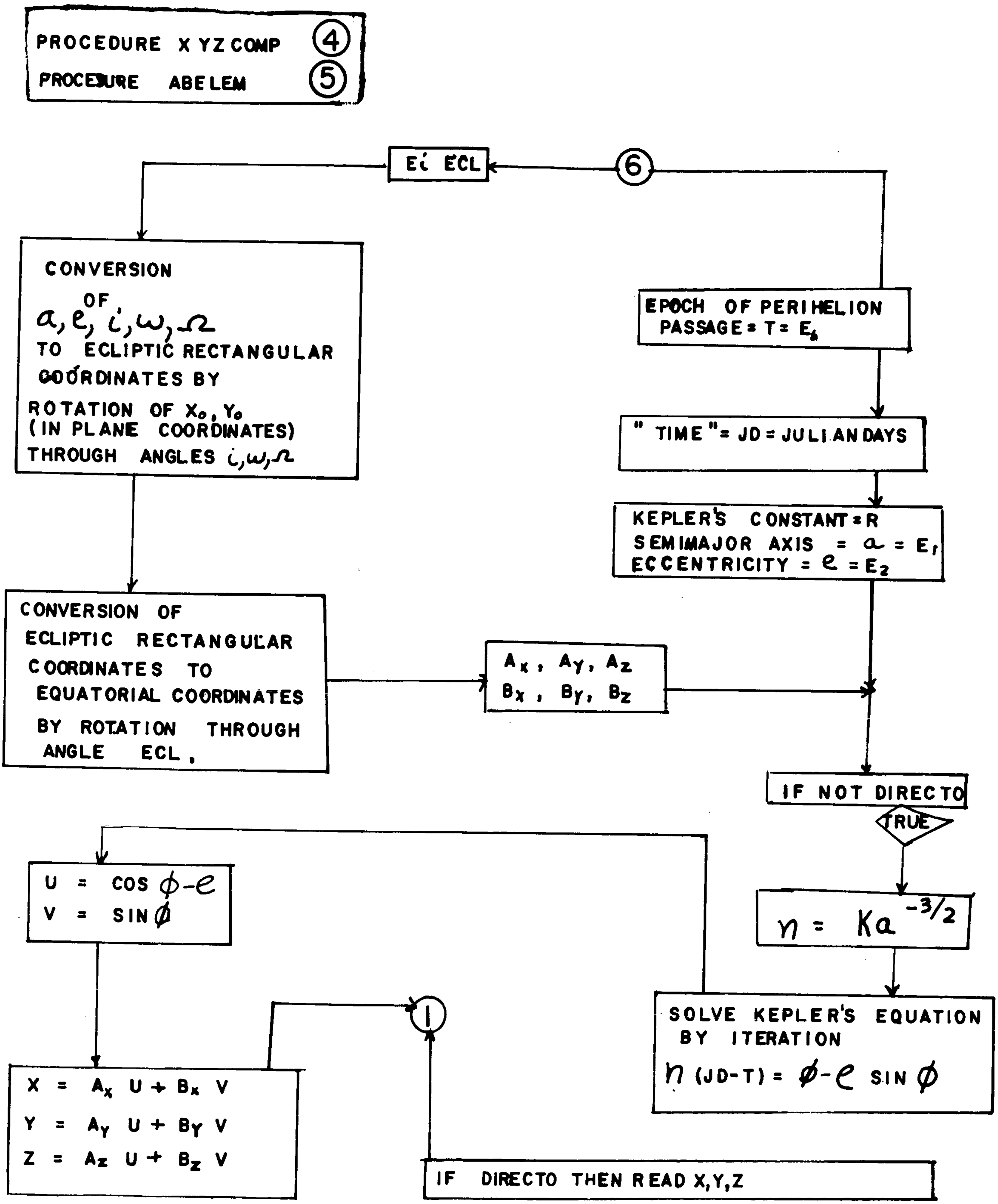
1
GEOCENTRIC SOLAR POSITIONS
GEOCENTRIC LUNAR POSITIONS
HELIOCENTRIC CENTER OF MASS (OF SOLAR SYSTEM) POSITIONS
12
CHANGE TO AU'S
J = 1 X - COORDINATE
J = 2 Y - COORDINATE
J = 3 Z - COORDINATE
INPUT (1,0,J)
INPUT (1,2,J)
INPUT (1,1,J)
INPUT (1,II,J) ← INPUT (1,2,J) — INPUT (1,1,J) — INPUT (1,0,J)
NUMERICAL DIFFERENTIATION OF INPUT (1,II,J)
X (1) = DOT (1,1), Y (1) = DOT (2,1), Z (1) = DOT (3,1)
11
HELIOCENTRIC PLANET POSITION
INPUT (1,0,J)
2
INPUT (1,0,J) ← INPUT (1,0,J) — INPUT (1,1,J) — INPUT (1,2,J)
NUMERICAL DIFFERENTIATION OF INPUT (1,0,J)
X1 (1) = VEL (1,0,1), Y1 (1) = VEL (2,0,1), Z1 (1) = VEL (3,0,1)
X = X(1) — VEL (1,0,1), Y = Y (1) — VEL (2,0,1), Z = Z (1) — VEL (3,0,1)
2
STATIONOCENTRIC COORDINATES (GEOMETRIC), EARTH, EQUALATORIAL, 1950.0 SYSTEM
X = INPUT (1,0,1), Y = INPUT (1,0,2), Z = INPUT (1,0,3)



2
1
STATIONOCENTRIC COORDINATES X, Y, Z (GEOMETRIC) (t)
R = X²+Y²+Z²
LIGHT TIME = τ
τ = R/(VELOCITY OF LIGHT)
CONVERT X, Y, Z BACK TO CENTER OF MASS COORDINATES (ADD ON COORDINATES OF SUN & CM)
INTERPOLATE TO (t - τ)
SUBTRACT OFF STATIONOCENTRIC COORDINATES OF SUN & CM
X', Y', Z' "APPARENT" COORDINATES
R₁² = X² + Y² + Z²
τ' = R₁/(VELOCITY OF LIGHT)
IF (τ - τ') > 10⁻¹²
τ ← τ'
FALSE
COORDINATES NOW CORRECTED FOR LIGHT TIME
1
Ẋ, Ẏ, Ż OBTAINED FROM 1
X, Y, Z FINAL = X' + Ẋτ', Y' + Ẏτ', Z' + Żτ'
APPARENT COORDINATES X, Y, Z NOW CORRECTED FOR STATIONOCENTRIC ABERRATION OF LIGHT
19

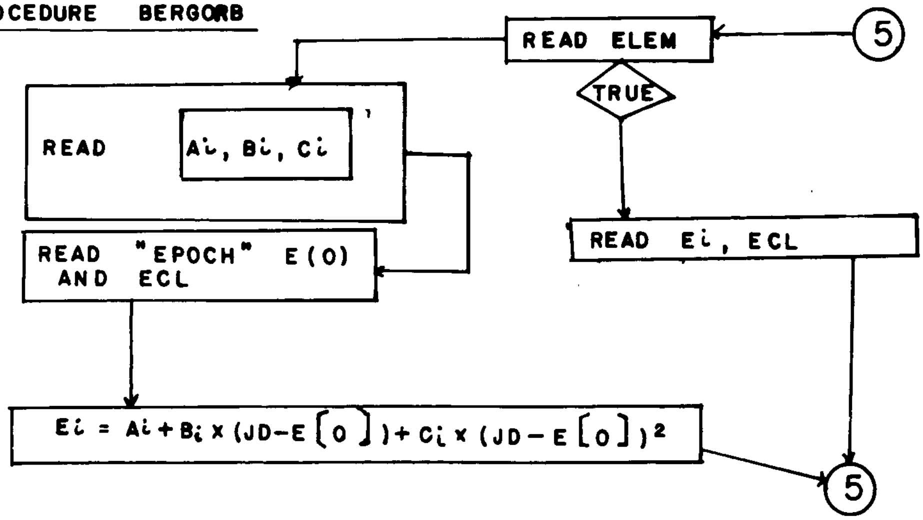
PROCEDURE FUNCTION;




PROCEDURE XYZCOMP ④
PROCEDURE ABELEM ⑤
Ei ECL
⑥
CONVERSION OF a, e, i, ω, Ω TO ECLIPTIC RECTANGULAR COORDINATES BY ROTATION OF X_0, Y_0 (IN PLANE COORDINATES) THROUGH ANGLES i, ω, Ω
EPOCH OF PERIHELION PASSAGE = T = E_6
" TIME " = JD = JULIAN DAYS
KEPLER'S CONSTANT = R
SEMIMAJOR AXIS = a = E_1
ECCENTRICITY = e = E_2
CONVERSION OF ECLIPTIC RECTANGULAR COORDINATES TO EQUATORIAL COORDINATES BY ROTATION THROUGH ANGLE ECL.
A_x, A_Y, A_Z
B_X, B_Y, B_Z
IF NOT DIRECTO
TRUE
$n = Ka^{-3/2}$
U = COS $\phi - e$
V = SIN ϕ
SOLVE KEPLER'S EQUATION BY ITERATION
$n(JD - T) = \phi - e \sin \phi$
X = A_X U + B_x V
Y = A_Y U + B_Y V
Z = A_Z U + B_Z V
①
IF DIRECTO THEN READ X,Y,Z

(6)

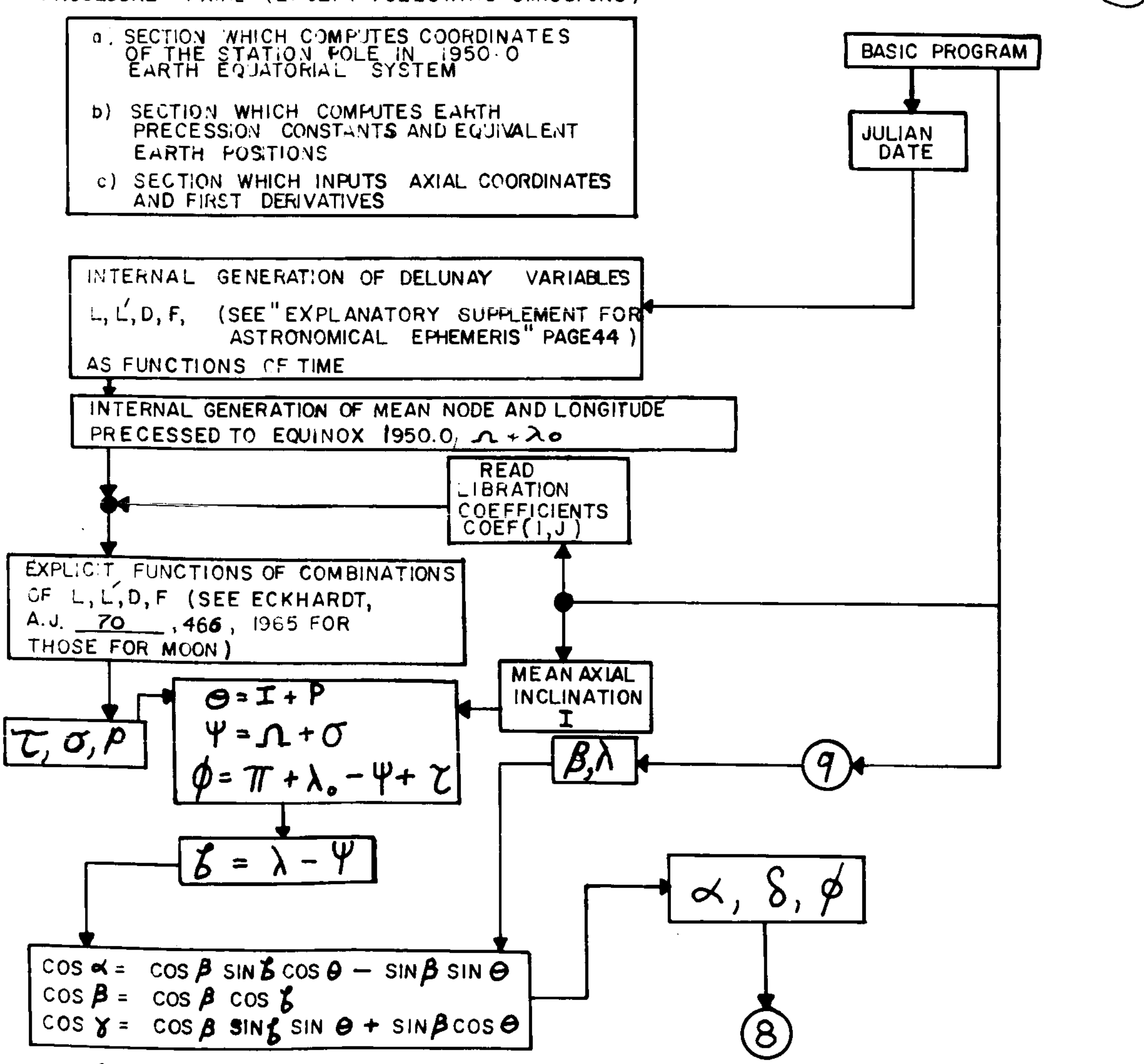
PROCEDURE BERGORB



NOTE: E_i ARE ORBITAL ELEMENTS $a, e, i, \omega, \Omega, T$.

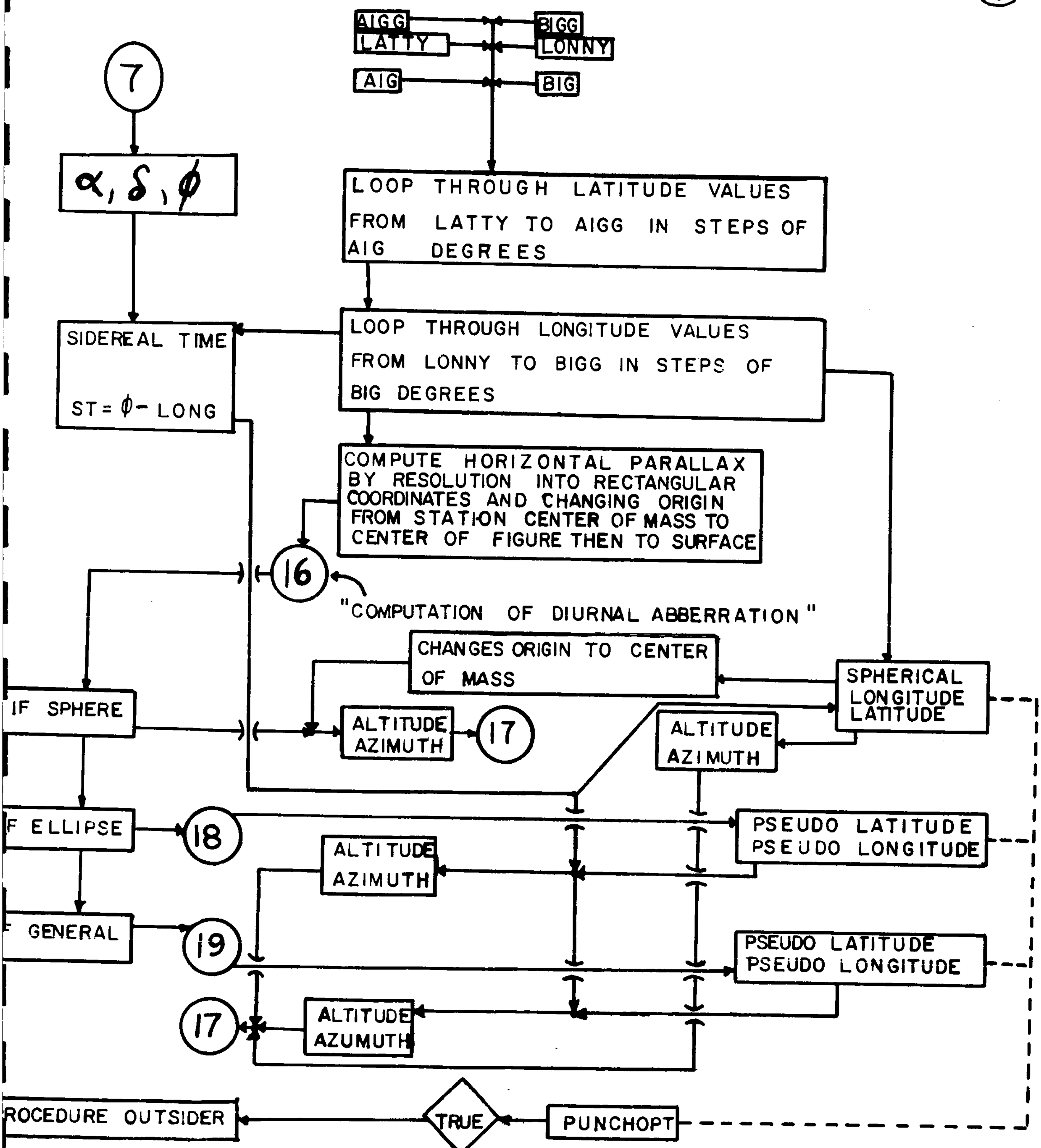
(7)

PROCEDURE AXIAL (EXCEPT FOLLOWING OMISSIONS)

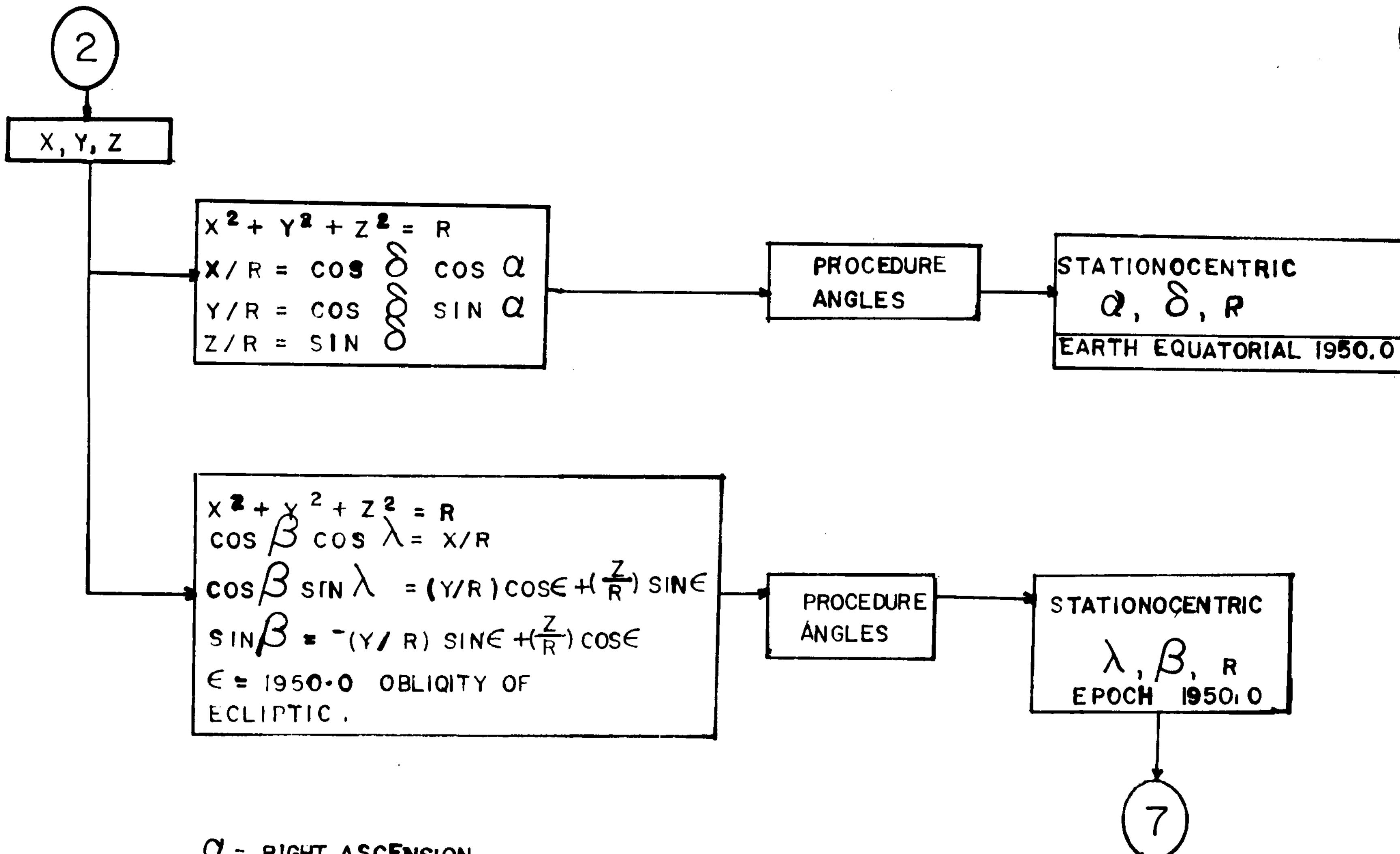


ϕ = ZERO MERIDIAN "SIDEREAL TIME"

PROCEDURE ALTAZ



9

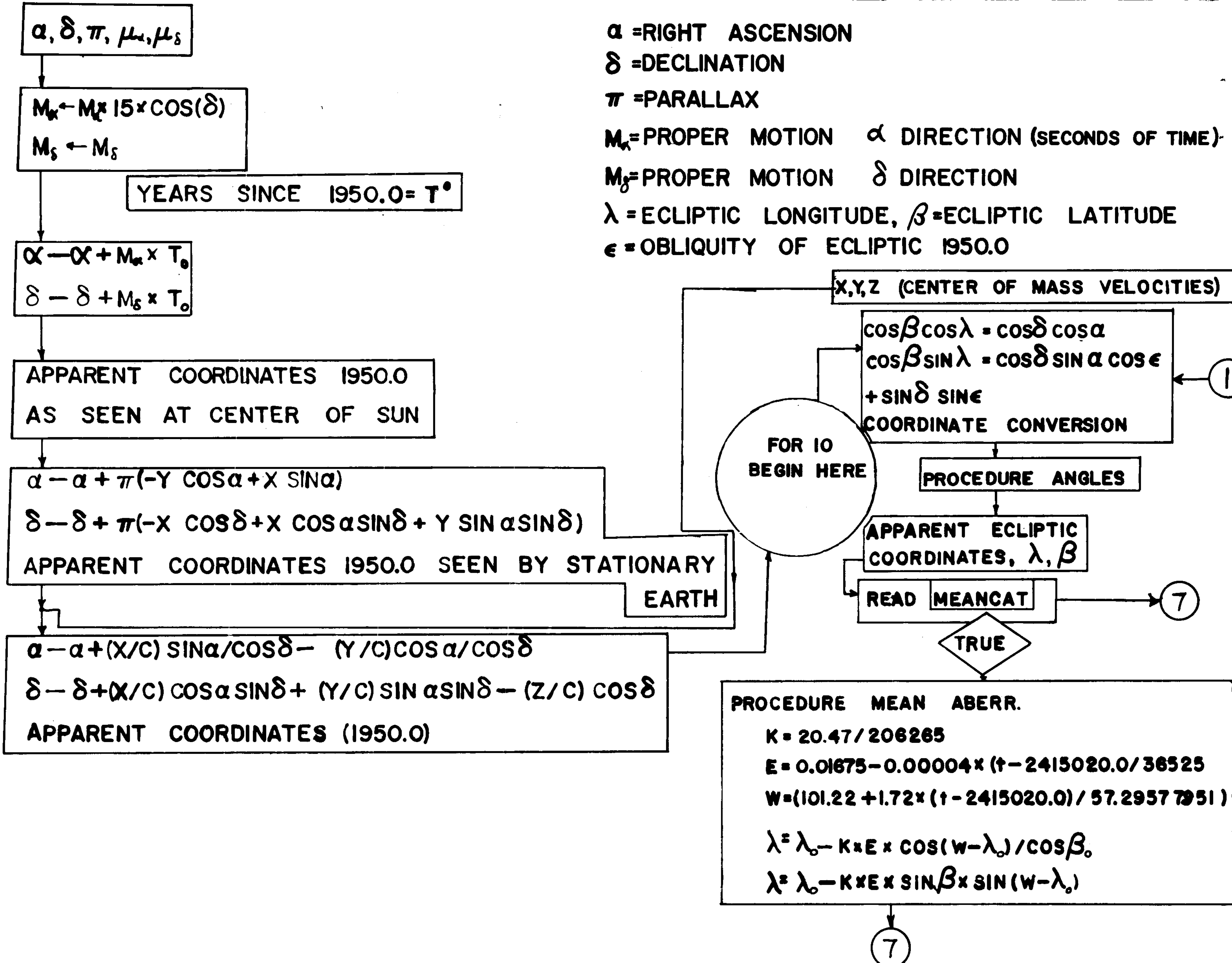


α = RIGHT ASCENSION

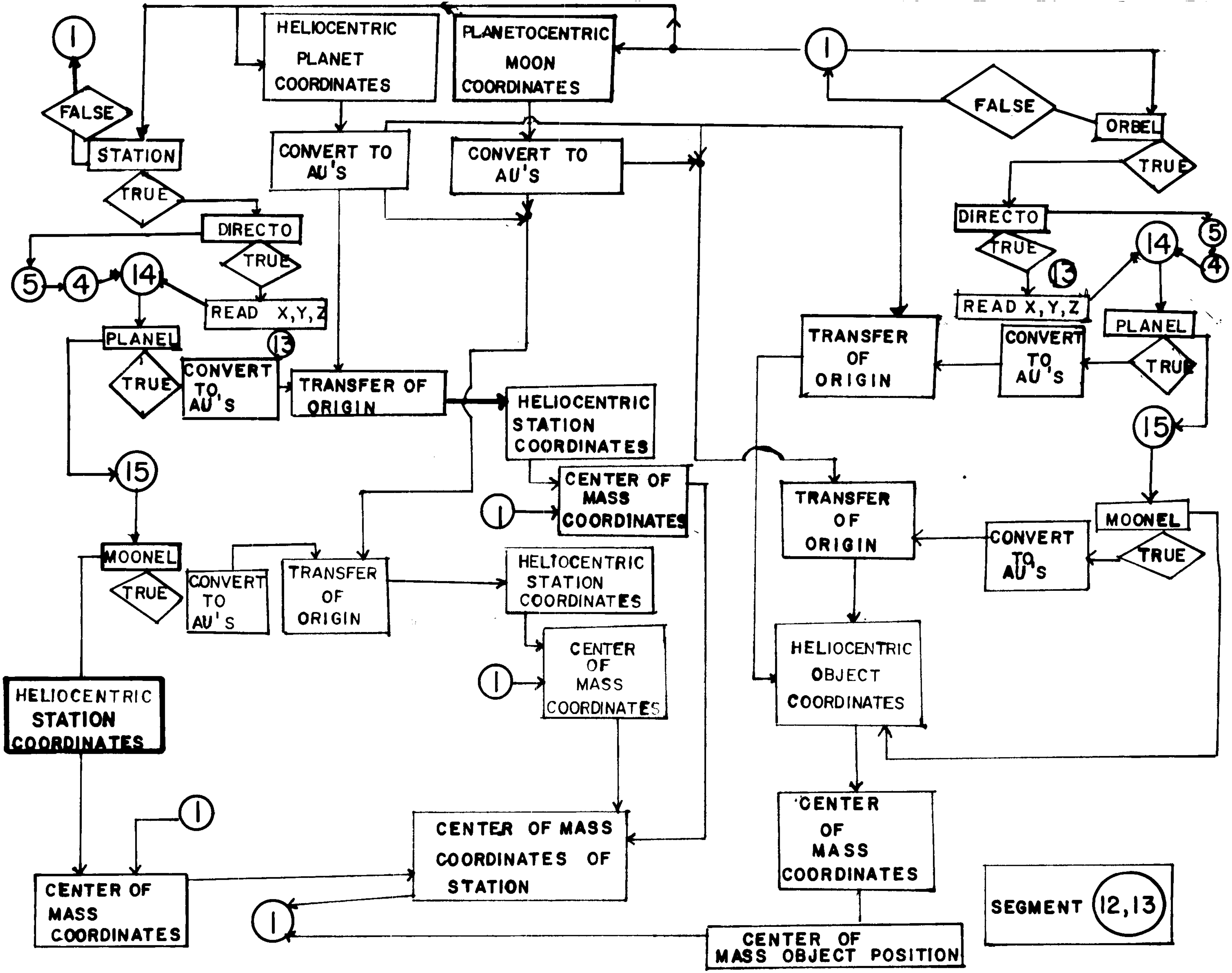
δ = DECLINATION

λ = ECLIPTIC LONGITUDE

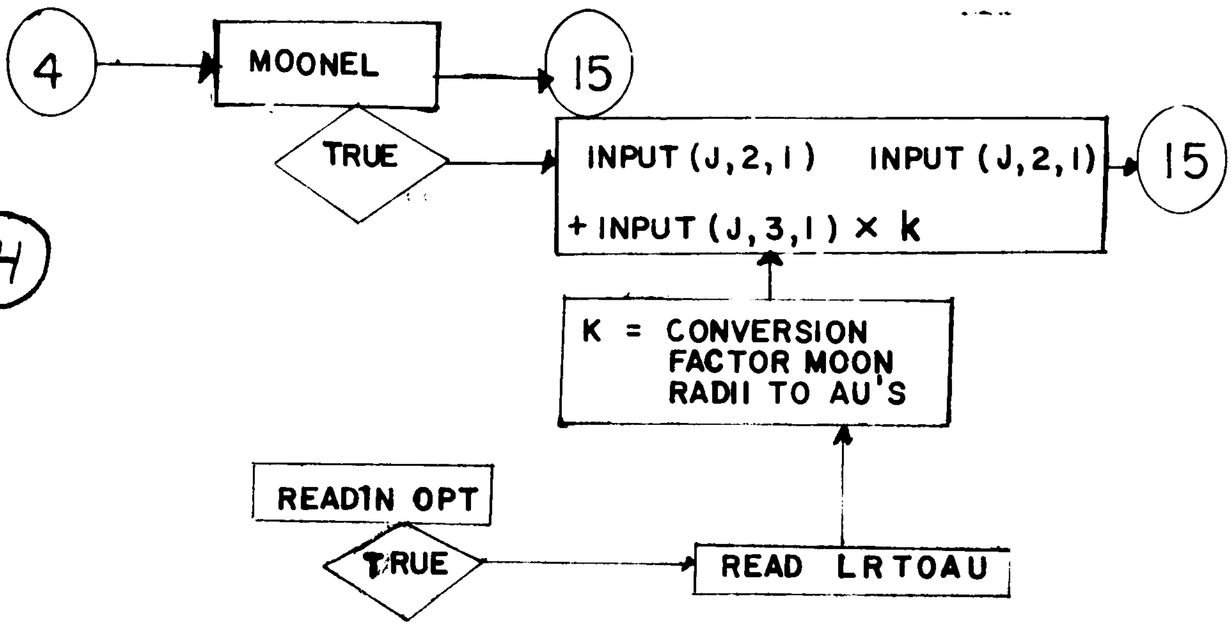
β = ECLIPTIC LATITUDE



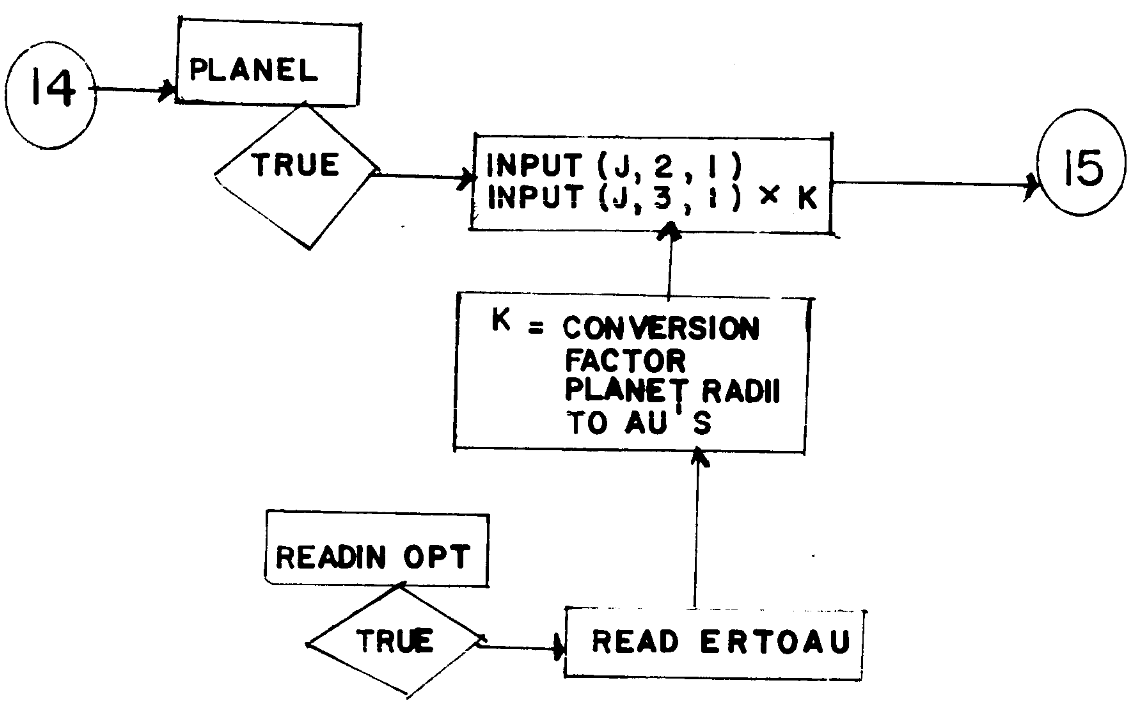
α, δ, π, μα, μδ
Mα ← Mα × 15 × COS(δ)
Mδ ← Mδ
YEARS SINCE 1950.0 = T°
α ← α + Mα × T₀
δ ← δ + Mδ × T₀
APPARENT COORDINATES 1950.0
AS SEEN AT CENTER OF SUN
α ← α + π(−Y COSα + X SINα)
δ ← δ + π(−X COSδ + X COSα SINδ + Y SINα SINδ)
APPARENT COORDINATES 1950.0 SEEN BY STATIONARY EARTH
α ← α + (X/C) SINα/COSδ − (Y/C) COSα/COSδ
δ ← δ + (X/C) COSα SINδ + (Y/C) SINα SINδ − (Z/C) COSδ
APPARENT COORDINATES (1950.0)
α = RIGHT ASCENSION
δ = DECLINATION
π = PARALLAX
Mα = PROPER MOTION α DIRECTION (SECONDS OF TIME)
Mδ = PROPER MOTION δ DIRECTION
λ = ECLIPTIC LONGITUDE, β = ECLIPTIC LATITUDE
ε = OBLIQUITY OF ECLIPTIC 1950.0
X,Y,Z (CENTER OF MASS VELOCITIES)
cosβ cosλ = cosδ cosα
cosβ sinλ = cosδ sinα cosε
+ sinδ sinε
COORDINATE CONVERSION
1
FOR 10
BEGIN HERE
PROCEDURE ANGLES
APPARENT ECLIPTIC
COORDINATES, λ, β
READ MEANCAT
7
TRUE
PROCEDURE MEAN ABERR.
K = 20.47/206265
E = 0.01675 − 0.00004 × (t − 2415020.0/36525
W = (101.22 + 1.72 × (t − 2415020.0)/57.2957 7951)
λ = λ₀ − K × E × COS(W − λ₀)/COSβ₀
λ = λ₀ − K × E × SINβ × SIN(W − λ₀)
7


1
FALSE
STATION
TRUE
DIRECTO
TRUE
5
4
14
READ X,Y,Z
PLANEL
TRUE
13
CONVERT TO AU'S
TRANSFER OF ORIGIN
15
MOONEL
TRUE
CONVERT TO AU'S
TRANSFER OF ORIGIN
HELIOCENTRIC STATION COORDINATES
CENTER OF MASS COORDINATES
HELIOCENTRIC PLANET COORDINATES
PLANETOCENTRIC MOON COORDINATES
CONVERT TO AU'S
CONVERT TO AU'S
HELIOCENTRIC STATION COORDINATES
CENTER OF MASS COORDINATES
HELIOCENTRIC STATION COORDINATES
CENTER OF MASS COORDINATES
CENTER OF MASS COORDINATES OF STATION
FALSE
ORBEL
TRUE
DIRECTO
TRUE
READ X,Y,Z
PLANEL
TRUE
CONVERT TO AU'S
TRANSFER OF ORIGIN
MOONEL
TRUE
CONVERT TO AU'S
TRANSFER OF ORIGIN
HELIOCENTRIC OBJECT COORDINATES
CENTER OF MASS COORDINATES
CENTER OF MASS OBJECT POSITION
SEGMENT 12,13

14


4
MOONEL
15
TRUE
INPUT (J,2,1) INPUT (J,2,1)
+ INPUT (J,3,1) × k
15
K = CONVERSION FACTOR MOON RADII TO AU'S
READIN OPT
TRUE
READ LRTOAU

15


14
PLANEL
TRUE
INPUT (J, 2, 1)
INPUT (J, 3, 1) × K
15
K = CONVERSION FACTOR PLANET RADII TO AU'S
READIN OPT
TRUE
READ ERTOAU

ANGULAR ROTATION RATE, W (WOMEGA)

7

IF EQUATORIAL VELOCITY IS GREATER THAN 0.001 KM/SEC

PROCEDURE DIURNCOMP

$$\Delta\alpha = (W \times R / C) \cos(HA) \cos LAT / \cos\delta$$

$$\Delta\delta = (W \times R / C) \sin(HA) \sin\delta \cos LAT$$

8

R = RADIUS OF BODY

C = SPEED OF LIGHT

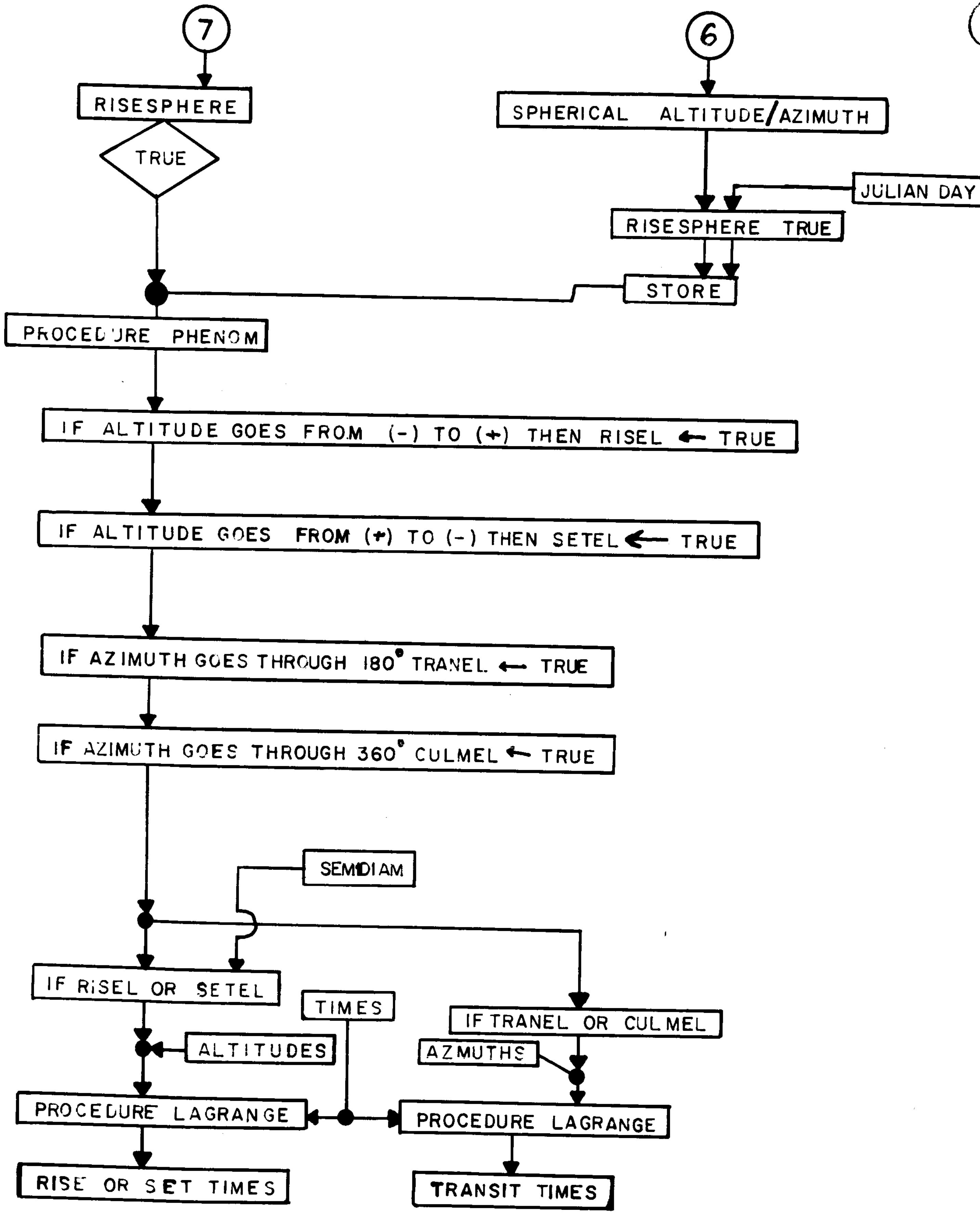
HA = LOCAL HOUR ANGLE

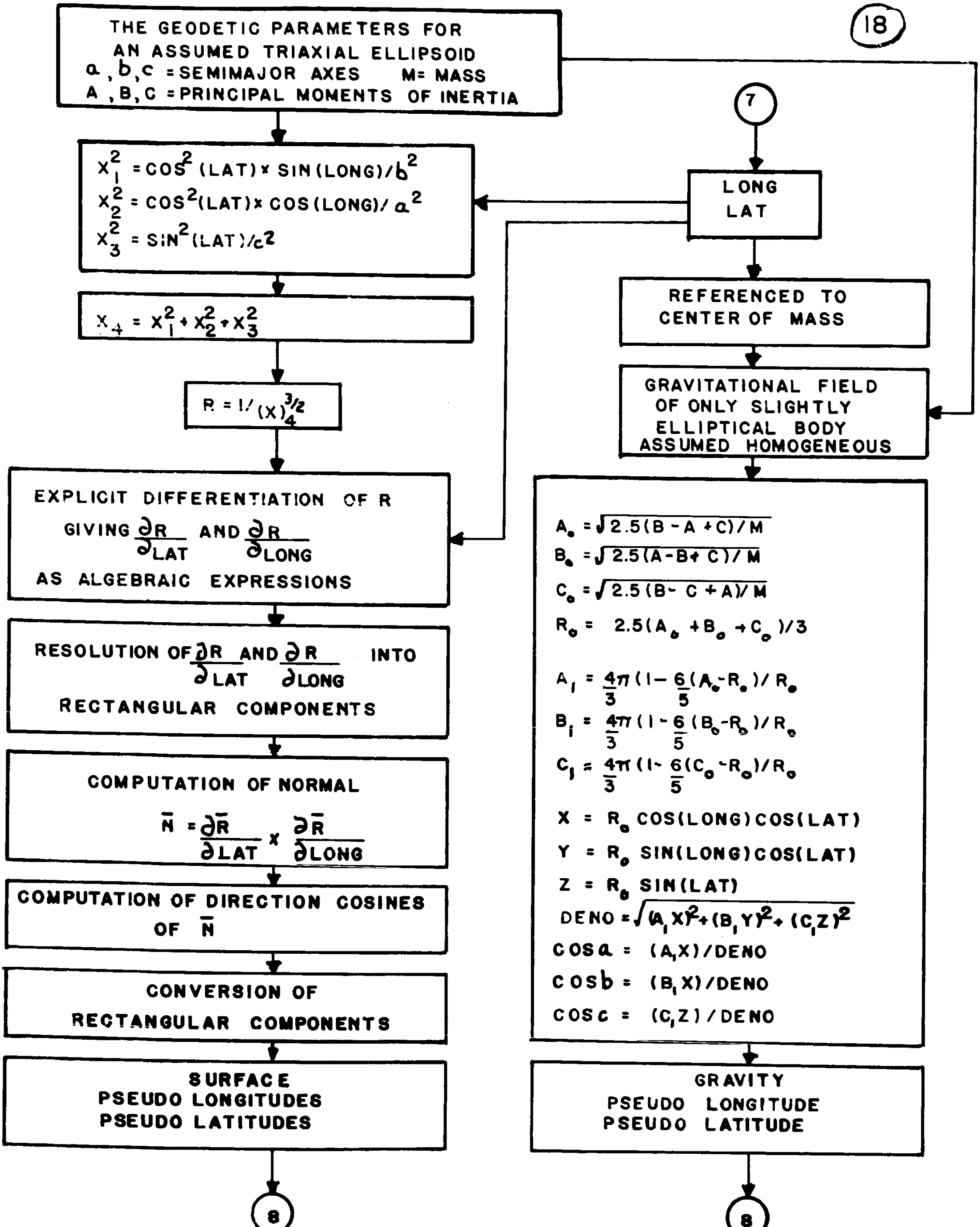
LAT = LATITUDE

α = RIGHT ASCENSION

δ = DECLINATION

W = ANGULAR ROTATION RATE


7
6
RISESPHERE
TRUE
SPHERICAL ALTITUDE/AZIMUTH
JULIAN DAY
RISESPHERE TRUE
STORE
PROCEDURE PHENOM
IF ALTITUDE GOES FROM (-) TO (+) THEN RISEL ← TRUE
IF ALTITUDE GOES FROM (+) TO (-) THEN SETEL ← TRUE
IF AZIMUTH GOES THROUGH 180° TRANEL ← TRUE
IF AZIMUTH GOES THROUGH 360° CULMEL ← TRUE
SEMIDIAM
IF RISEL OR SETEL
TIMES
IF TRANEL OR CULMEL
ALTITUDES
AZMUTHS
PROCEDURE LAGRANGE
PROCEDURE LAGRANGE
RISE OR SET TIMES
TRANSIT TIMES


THE GEODETIC PARAMETERS FOR AN ASSUMED TRIAXIAL ELLIPSOID
a, b, c = SEMIMAJOR AXES   M = MASS
A, B, C = PRINCIPAL MOMENTS OF INERTIA
7
LONG
LAT
$X_1^2 = COS^2(LAT) \times SIN(LONG)/b^2$
$X_2^2 = COS^2(LAT) \times COS(LONG)/a^2$
$X_3^2 = SIN^2(LAT)/c^2$
$X_4 = X_1^2 + X_2^2 + X_3^2$
$R = 1/(X)_4^{3/2}$
REFERENCED TO CENTER OF MASS
GRAVITATIONAL FIELD OF ONLY SLIGHTLY ELLIPTICAL BODY ASSUMED HOMOGENEOUS
EXPLICIT DIFFERENTIATION OF R GIVING $\frac{\partial R}{\partial LAT}$ AND $\frac{\partial R}{\partial LONG}$ AS ALGEBRAIC EXPRESSIONS
RESOLUTION OF $\frac{\partial R}{\partial LAT}$ AND $\frac{\partial R}{\partial LONG}$ INTO RECTANGULAR COMPONENTS
COMPUTATION OF NORMAL
$\bar{N} = \frac{\partial \bar{R}}{\partial LAT} \times \frac{\partial \bar{R}}{\partial LONG}$
COMPUTATION OF DIRECTION COSINES OF $\bar{N}$
CONVERSION OF RECTANGULAR COMPONENTS
SURFACE
PSEUDO LONGITUDES
PSEUDO LATITUDES
8
$A_o = \sqrt{2.5(B - A + C)/M}$
$B_o = \sqrt{2.5(A - B + C)/M}$
$C_o = \sqrt{2.5(B - C + A)/M}$
$R_o = 2.5(A_o + B_o + C_o)/3$
$A_1 = \frac{4\pi}{3}(1 - \frac{6}{5}(A_o - R_o)/R_o$
$B_1 = \frac{4\pi}{3}(1 - \frac{6}{5}(B_o - R_o)/R_o$
$C_1 = \frac{4\pi}{3}(1 - \frac{6}{5}(C_o - R_o)/R_o$
$X = R_o$ COS(LONG) COS(LAT)
$Y = R_o$ SIN(LONG) COS(LAT)
$Z = R_o$ SIN(LAT)
$DENO = \sqrt{(A_1 X)^2 + (B_1 Y)^2 + (C_1 Z)^2}$
$COS\,a = (A_1 X)/DENO$
$COS\,b = (B_1 X)/DENO$
$COS\,c = (C_1 Z)/DENO$
GRAVITY
PSEUDO LONGITUDE
PSEUDO LATITUDE
8

Appendix 2 - SUPPLEMENTARY REFERENCES.

ADDITIONAL REFERENCES

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Appendix 3 - TABLE OF SYMBOLS.

TABLE OF SYMBOLS USED IN LANS.

i, Δ, Ω', I - lunar equatorial constants relative to earth equator (used in American Ephemeris and Nautical Almanac).

A, B, C - Principal Moments of Inertia (Moon).

α, β, γ - Moments of Inertia ratios (Moon).

$f = -\alpha/\beta$ - Lunar Flattening ratio.

$\Delta\theta$ - Total Deflection of the Vertical.

X_o, Y_o, Z_o - Coordinates of Center-of-Mass of Solar System, (heliocentric equatorial 1950.0).

M_n - Mass Fraction relative to Sun of nth planet. X_n, Y_n, Z_n - Heliocentric rectangular equatorial 1950.0 coordinates of nth planet.

X, Y, Z or $\overline{X}$, $\overline{Y}$, $\overline{Z}$ - Geocentric or planetocentric coordinates (1950.0).

X, Y, Z or $\overline{X}$, $\overline{Y}$, $\overline{Z}$ - heliocentric coordinates (1950.0).

A_x, A_y, A_z, B_x, B_y, B_z - Equatorial orbital elements (see Dubyago 1962).

a, e, ω, i, Ω, T - "Normal" orbital elements:

a = semi-major axis
e = eccentricity
ω = argument of periapsis
i = inclination to ecliptic
Ω = ascending node
T = time of periapsis

t_o - Time epoch of geometric ephemeris.

τ - light time.

M_o - Mean catalogue place.

A - Apparent coordinate position.

R - Complete star reduction = apparent = mean.

E - "E" term of aberration (see Explanatory Supplement).

λ, β - Ecliptic longitude and latitude.

K - Aberration constant.

e, ω - orbital eccentricity (of earth), argument of perihelion of earth.

$\mho$ - Longitude (ecliptic) of the descending node (orbital or equatorial).

Ω - Longitude (ecliptic) of the ascending node. (orbital or equatorial).

W - angular rotation rate of body (sometimes in literature as ω).

α, δ (or R.A., Dec.) - Equatorial angular coordinates RIGHT ASCENSION DECLINATION

i - Inclination of orbit or equatorial plane to ecliptic.

L. H. A. - Local Hour Angle of object (sometimes in literature as left hour angle.)

HA - Hour angle (zero longitude or local).

Long. Lat. - Longitude and latitude, topocentric station coordinates.

ST - Sidereal time (zero longitude or local).

X_1, Y_1, Z_1 - Heliocentric rectangular coordinates.

X_2, Y_2, Z_2 - Selenocentric rectangular coordinates.

AZ, ALT - Topocentric angular coordinates of object.

X', Y', Z' - Heliocentric or geocentric <u>ecliptic</u> rectangular coordinates.

ε - Obliquity of ecliptic (= i for the earth).

$\overline{R}$ - Geocentric vector of sun.

$\overline{r}$ - Geocentric vector of moon.

$\overline{p}$ - Selenocentric vector of sun.

$\overline{r}''_p$ - Selenocentric vector of planet.

$\overline{r}_p$ - Geocentric vector of planet.

$\overline{r}'_p$ - Heliocentric vector of planet.

$\overline{r}_{cm}$ - Center-of-mass vector (heliocentric).

$(-\overline{p})$ - Heliocentric vector of station.

ϕ - Total aberration of light.

ϕ_s - Stellar aberration.

ϕ_p - Planetary aberration.

σ, ρ, τ - Lunar libration components	τ is the perturbation of sidereal time σ is the perturbation of the node ρ is the nutation component.
W, I. - Instantaneous lunar equatorial constants	W = rotation rate I = inclination, = longitude of descendent node.
L, L', F, D - Delunay Lunar Variables	L = mean anomaly of the moon L' = mean anomaly of the sun F = distance of moon from the node D = mean elongation of moon from sun.

K planet - Keplerian constant for a planet.

K sun - Keplerian constant for the sun.